Introduction to Probability

Math 30530, Section 01 — Fall 2012

Homework 2 — due Friday September 7

General information: Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread and revise your solutions until they are correct and concise. This will help deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

I plan to quickly post solutions to all the problems after I've collected them up.

Reading:

- Chapter 3
- Chapter 4
- Chapter 5

Problems: (**GW** indicates that the problem is taken from the course textbook by Gundlach and Ward)

1. For events A and B with $\Pr(A) > 0$, $\Pr(B) > 0$, we say that A is *positively correlated* with B if $\Pr(A|B) > \Pr(A)$ (in other words, B occurring *increases* the chance that A occurs). If $\Pr(A|B) < \Pr(A)$ then A is *negatively correlated* with B. (If $\Pr(A|B) = \Pr(A)$ then A is *not correlated* with B.)

   (a) Show that if A is positively correlated with B, then B is positively correlated with B; in other words, show that if $\Pr(A|B) > \Pr(A)$ then $\Pr(B|A) > \Pr(B)$. (The same holds for negatively correlated and uncorrelated; so we can from on just say "A and B are positively correlated", etc.)

   (b) Andy Murray and Roger Federer play a best-of-*five*-set tennis match. Each player has a .5 chance of winning each set, with each set independent of the others. Are the events "Federer wins the match (gets to three sets before Murray does)" and "Federer has a two sets to one advantage after three sets" positively correlated, negatively correlated, or uncorrelated?

   (c) I roll two dice and note the numbers that come up. Are the events "The sum of the numbers coming up is even" and "The two numbers are within one of each other" positively correlated, negatively correlated, or uncorrelated?

2. In one variant of the game of craps, a player rolls dice. If the sum is 7 or 11, he wins immediately. If the sum is 2 or 12, he loses immediately. If he rolls anything else (3, 4, 5, 6, 8, 9, 10), then that becomes his "point", and he repeatedly rolls the pair of dice until EITHER he gets his point again, in which case he wins, OR he rolls 7 or 11, in which case he losses.

   Suppose a player's point is 5. What is the probability that he wins eventually? In other words, when rolling a pair of dice repeatedly, and each time noting the sum of the numbers that come up, what is the probability that the sum of 5 will come up before either 7 or 11? (**Hint**: Section 3.4 of the textbook!)

3. Show that if $A_1, A_2, \ldots, A_n$ are independent events, then

$$\Pr(A_1 \cup A_2 \cup \ldots \cup A_n) = 1 - (1 - \Pr(A_1))(1 - \Pr(A_2)) \ldots (1 - \Pr(A_n))$$

4. ~~**GW** 4.1~~ (removed because the typos in the question are a little bit too much to correct without causing further confusion!)

5. **GW** 4.5

6. **GW** 4.6

7. **GW** 4.8 (just a, b, c)

8. **GW** 4.10

9. **GW** 4.15 (for this question and the next, assume that when a baby is born (s)he is equally likely to be a boy or a girl, and the genders of a couples' children are independent)
10. **GW** 4.16
11. **GW** 4.17
12. **GW** 5.2
13. **GW** 5.9
14. **GW** 5.11
15. **GW** 5.13
16. **GW** 5.19
17. **GW** 5.23
18. **GW** 5.24
19. **GW** 5.28