1. 55% of students read the Observer daily, 25% live off-campus, and 63% either live off campus or read the Observer daily (or both). I pick a student at random (all students equally likely). What is the probability that the student I pick BOTH lives on campus AND reads the Observer daily?

**Solution:** Let $R =$ reading observer, and $OC =$ living off-campus. Since

$$\Pr(OC \cup R) = \Pr(OC) + \Pr(R) - \Pr(OC \cap R)$$

we have $.63 = .25 + .55 - \Pr(OC \cap R)$, or $\Pr(OC \cap R) = .17$.

We are looking for $\Pr(OC^c \cap R)$, which is $\Pr(R) - \Pr(OC \cap R) = .55 - .17 = .38$.

**Note:** It is not correct to say $\Pr(OC^c \cap R) = \Pr(OC^c) \Pr(R)$; we don’t know that the events of living off-campus and reading the observer are independent.

2. Use the definition of conditional probability and the three basic rules of probability to show that for any two events $A$ and $B$ (with $\Pr(B) > 0$),

$$\Pr(A^c|B) = 1 - \Pr(A|B).$$

**Solution:** $\Pr(A^c|B) = \Pr(A^c \cap B)/\Pr(B)$ and $\Pr(A|B) = \Pr(A \cap B)/\Pr(B)$ by definition, so we want to show

$$\frac{\Pr(A^c \cap B)}{\Pr(B)} = 1 - \frac{\Pr(A \cap B)}{\Pr(B)}.$$

Multiplying through by $\Pr(B)$, this becomes $\Pr(A^c \cap B) = \Pr(B) - \Pr(A \cap B)$ or $\Pr(B) = \Pr(A^c \cap B) + \Pr(A \cap B)$. This is true because $B$ is the disjoint union of $A^c \cap B$ and $A \cap B$, and one of the basic rules of probability is that the probabilities of disjoint events add.

**Note 1:** After reducing to (1), it is not correct to say $\Pr(A^c \cap B) = \Pr(A^c) \Pr(B)$ and $\Pr(A \cap B) = \Pr(A) \Pr(B)$, and cancel the $\Pr(B)$’s to reduce to $\Pr(A^c) + \Pr(A) = 1$: we don’t know that the events $A$ and $B$ are independent.

**Note 2:** Some people reasoned out the truth intuitively; I docked points for this because the question specifically asked for a proof using the definitions and basic rules.

3. I toss a dime and a nickel. Let $A$ be the event that both coins come up showing the same side (either both heads or both tails). Let $B$ be the event that the dime comes up tails.

(a) Are $A$ and $B$ mutually exclusive?

**Solution:** Listing the dime first, $A = \{HH, TT\}$ and $B = \{TH, TT\}$, so $A \cap B = \{TT\} \neq \emptyset$, so $A$ and $B$ are not mutually exclusive.
(b) Are $A$ and $B$ independent?

**Solution:** $Pr(A) = 2/4 = 1/2$, $Pr(B) = 2/4 = 1/2$ and $Pr(A \cap B) = 1/4$. So $Pr(A \cap B) = Pr(A) Pr(B)$, and the events are independent.