Math 30530 — Introduction to Probability

Puzzler 3

Solutions

The puzzle:

Roll six dice, and count the number of different numbers that are face-up; let X be this number. The possible values for X are 1 through 6. The puzzle was to determine whaich is the most likely value that X takes on, and whether you would be happy betting on that number coming up; i.e., whether that most common number comes up more than half the time.

Solution:

Considering the six dice as distinct, there are $6^6 = 46,656$ possible outcomes of rolling the dice. We'll count in how many ways we get **four** different numbers.

There are $\binom{6}{4} = 15$ different choices of four numbers to come up; say 1, 2, 3 and 4.

One way to get 1,2,3,4 is to get three of one number and one of each of the others. For example, we could three 1's, one 2, one 3 and one 4. There are $\binom{6}{3}$ ways of choosing the three dice that show 1, and then 3! ways of deciding which of the remaining three dice show 2, 3 and 4. That's 120 ways; but since we could also have had three 2's, three 3's or three 4's, thats 480 ways.

Another way to get 1,2,3,4 is to get two of one number, two of another, and one of each of the other two. There are $\binom{4}{2}$ ways of choosing which are the two repeated numbers; say for example we have two 1's, two 2's, one 3 and one 4. There are $\binom{6}{2}$ ways of choosing which dice show 1's; subsequently $\binom{4}{2}$ ways of choosing which dice show 2's; and subsequently 2 ways of settling the last two numbers. That's $\binom{4}{2}\binom{6}{2}\binom{4}{2}2 = 1080$ ways in all of having a 2-2-1-1 split.

That gives $1080 + 480 = 1560$ ways of getting 1, 2, 3, 4; so $1560 \times 15 = 23,400$ ways of getting four different numbers.

Since $23,400$ is (just) more that half of $46,656$, this is the most likely number of different numbers. And since the probability of getting four different numbers is (barely) more than .5, the bet is in your favor.

Fully correct solvers:

- Michael MacGillivray (1,4)
- Roisin McCord (2,5 — winner!)
- John Macke (3,6)