Math 30530: Introduction to Probability, Fall 2013

Midterm Exam I — make-up exam

Monday, October 15

Name: ____________________________ **Instructor:** David Galvin

This exam contains 8 problems on 7 pages (including the front cover).
Calculators may be used, but no books or notes.
Show all your work on the paper provided.
The honor code is in effect for this exam.

Scores

Question	Score	Out of
1 — 5		20
6		10
7		10
8		10
Total		50

GOOD LUCK !!!

1. A and B are events in a probability space. Which one of the following statements is NOT true?

   (a) If $A \subset B$ then $\Pr(A) \leq \Pr(B)$

   (b) $\Pr(A \cap B) \geq \Pr(A) + \Pr(B) - 1$

   (c) If $\Pr(B) > 0$ and $\Pr(A) > 0$ then $\Pr(A|B) = \Pr(A)$

   (d) $\Pr(A \cap B^c) = \Pr(A \cup B) - \Pr(B)$.

2. I have m bins in front of me, and n balls beside me. One after another I throw the balls into the bins, with each ball equally likely to fall into each bin, and all throws independent. What is the probability, at the end of all this, that the ith bin is empty (for some particular i)?

   (a) $\left(1 - \frac{1}{m}\right)^n$

   (b) $\left(1 - \frac{1}{n}\right)^m$

   (c) $\binom{m}{n}\left(1 - \frac{1}{n}\right)^m$

   (d) $\binom{m}{n}\left(1 - \frac{1}{m}\right)^n$.

3. A well-shuffled deck of cards is dealt evenly (26 cards each) between two people, Andy and Bette. What is the probability that Andy gets all four aces?

   (a) $\frac{\binom{48}{22}}{\binom{52}{26}}$

   (b) $4\frac{\binom{48}{22}}{\binom{52}{26}}$

   (c) $\frac{48!}{22!}\frac{52!}{26!}$

   (d) $4!\frac{\binom{48}{22}}{\binom{52}{26}}$.

4. I have a coin which comes up Heads 40% of the time. I toss it repeatedly (and independently), keeping track of the number of times I have tossed a Head, and I stop when I have reached exactly 4 Heads. What is the probability that I stop at exactly the 7th toss?

   (a) .193536

   (b) .05184

   (c) .110592

   (d) .2734375.

5. I flip a fair coin five times. Alice counts X, the number of Heads that occurred in the first three tosses, and Bob counts Y, the number of Tails that occurred in the last three tosses. Calculate the value of the joint mass function of X and Y, $p_{X,Y}(x, y)$, when $x = 2$ and $y = 1$.

   (a) 9/64
   (b) 1/8
   (c) 9/32
   (d) 5/32.

6. (a) Use the rules of probability to show that if E and F are any events in a probability space, with $\Pr(F) > 0$, then $\Pr(E|F) + \Pr(E^c|F) = 1$.

(b) A woman gives birth to a child in the hospital's maternity ward. After a while, the child is brought into the hospital's nursery. Before the child is brought in, there are 5 boys and 10 girls in the nursery. Sometime after the child is brought in, a doctor walks into the nursery, picks a child at random, and notices that it's a boy. What's the probability that the woman gave birth to a boy? (You should assume that when a child is born, the probability that it is a boy is .5 and the probability that it is a girl is .5.)

(c) In the same scenario, what's the probability that the woman gave birth to a girl?

7. (a) Show that if A and B are independent events in a probability space, then A and B^c are also independent.

(b) A fair coin is tossed n times. Let E be the event that both heads and tails occur, and let F be the event that at most one head occurs.

i. If $n = 2$, are the events E and F independent? Justify your answer.

ii. If $n = 3$, are the events E and F independent? Justify your answer.

8. The St. Joseph County superior court has two court reporters, Mr. Jakes and Mr. Williams. On average, Mr. Jakes makes 2 typographic errors per page that he types, and Mr. Williams makes on average 5 errors per page.

   (a) On a day when Mr. Jakes is working, I examine a randomly chosen page of the court report. What is the probability that I see at least 1 error? (I am perfect at spotting errors.)

   (b) Mr. Jakes works three days a week, and Mr. Williams works the other two. On a randomly chosen day, I inspect a randomly chosen page of the court report. What is the probability that I find exactly 2 errors?