Math 30530: Introduction to Probability, Fall 2013

Practice Final Exam

(Slightly modified from actual Fall 2012 final)

1. A calipers makes measurements which have an error that is normally distributed with mean 0 and variance σ^2 (which can be controlled by the user, at a cost).

   (a) What value should σ^2 be set to, to be 98% certain that the measured value is within ± 3mm of the actual value?

   (b) I set σ^2 to 4, take nine measurements, and average them (so the error I make is the average of nine independent errors). What is the probability that this average error is no more than ± 1?

2. *Two pairs* in poker is a collection of 5 cards with two of one kind, two of another different kind, and a fifth card of a different kind again. (For example: $2\heartsuit, 2\clubsuit, 7\spadesuit, 7\heartsuit, K\clubsuit$.)

   (a) I select 5 cards at random from a shuffled regular deck of 52 cards. What is the probability of getting two pairs (as described above)?

   (b) After having selected the 5 cards, I turn over three of them and find that they are the ace and two of spades and the two of clubs ($A\spadesuit, 2\spadesuit, 2\clubsuit$). Given this information, what is now the probability that I have gotten two pairs?

3. I toss two coins, and let X be the number that come up heads. I then re-toss all the coins that originally came up *tails*, and let Y be the number of those that come up heads on the re-toss.

   (a) Compute $\Pr(X = 1, Y = 1)$.

   (b) Fill in the values in the table below to give the joint mass function of X and Y.

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$			
$Y = 1$			
$Y = 2$			

(c) Compute $\mathrm{Cov}(X, Y)$.

4. Suppose X is uniform on the interval $[a, b]$ $(0 < a < b)$. Let $Y = 1/X$.

   (a) Compute the cumulative distribution function of Y.

   (b) Compute the density function of Y.

   (c) Compute the expected value of Y and the expected value of Y^2.

5. Let X and Y be continuous random variables with joint density

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{x} & \text{if } 0 < y \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Compute $E(X)$.

   (b) Write down (no need to evaluate) an integral whose value is $\Pr(X - Y \geq 1/2)$.

   (c) Compute the marginal density of Y.

6. For each of the following scenarios, say what random variable you would use to model the situation (for example: exponential, or binomial, etc.), what the value of the parameter or parameters should be, and what sum or integral you would set up to answer the question. **DON'T CALCULATE THE SUM OR INTEGRAL!**

   (a) When playing the piano, Glenn Gould breaks into a momentary hum on average once every 7 minutes. How likely is it that I have to listen to him play for more than 30 minutes to hear him break into a momentary hum for the first time?

   (b) 5000 students from school N and 1000 students from school M enter a lottery. 2500 names from among the 6000 are selected to be the winners. What is the probability that between 400 and 440 (inclusive) of the winners are from school M?

   (c) On average there are three shooting stars a night visible from South Bend. I watch the skies closely for a total of three nights. What's the probability that you see between 6 and 8 shooting stars (inclusive)?

   (d) There are 600 frogs in a swamp. On average, each frog leaps out of the swamp to catch a fly 16 times per 8-hour night. If I watch the swamp for one minute during the night, what's the probability that I see at least 16 frogs leaping for flies?

7. When a Geiger counter is waved over a lump of beryllium-11, it makes clicks at a constant average rate of one click per 15 seconds.

   (a) I have just heard a click. What is the probability that I wait between 10 and 20 seconds to hear the next click? (Just write down an integral; no need to evaluate it.)

(b) Write down a sum (no need to evaluate it!) that equals the probability that the Geiger counter clicks between 200 and 240 times (inclusive) over the course of 1 hour.

8. Experience has shown that when I am writing reports in the morning, I tend to make on average 1 typo per page. When I write in the afternoon, I make on average a .5 typos per page (1 per two pages). On report-writing days, I write four pages in the morning and five in the afternoon.

   (a) Compute the probability that I make 2 or fewer typos in the morning. (Here and in the following parts, you don't need to simplify your answer.)

   (b) What is the probability that I make 2 or fewer typos during the whole day?

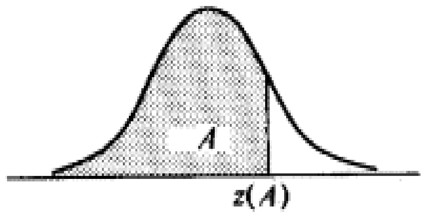
   (c) The day after report-writing day, I pick up a random report page and find two typos. What is the probability that I wrote that page in the morning?

9. Suppose that 70% of the families in your (very large) city have no dogs, 22% have 1 dog and 8% have 2 dogs.

   (a) Let X be the number of dogs that a randomly chosen family has. Compute $E(X)$ and $\mathrm{Var}(X)$.

   (b) Assuming your 200 family neighborhood constitutes a random sample and that families make their choices about dog ownership independently, approximate (using Central Limit Theorem) the probability that there are more than 90 dogs in your neighborhood.

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998