Introduction to Probability, Fall 2013

Math 30530 Section 01

Homework 1 — due in class Friday, September 6

General information

Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread and revise your solutions until they are correct and concise. This will help deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

At the top of the first page, you should write your name, the course number and the assignment number. If you use more than one page, you should staple all your pages together. The grader reserves the right to leave ungraded any assignment that is disorganized, untidy or incoherent.

I plan to post solutions to all the problems after I've collected them up.

Reading

- Introduction to Chapter 1
- Sections 1.1, 1.2 and 1.3

Problems

1. Chapter 1, problem 1.
2. Chapter 1, problem 2 (**Note**: I don't expect you to remember these kinds of equations. The point of this exercise is just to highlight that the same event can often be expressed in lots of different ways).
3. Chapter 1, problem 5.
4. Chapter 1, problem 6.

5. (a) Chapter 1, problem 7.
   (b) Let E_n denote the event that the experiment described in Chapter 1, problem 7 ends after exactly n rolls.
      i. What points of the sample space are contained in E_n?
      ii. What points of the sample space are contained in $(\cup_{n=1}^{\infty} E_n)^c$?

6. Chapter 1, problem 8 (so that everyone is using the same notation: say that A, B and C are the three opponents, and that you know that you can beat A with probability p, B with probability q and C with probability r; assume that $p > q > r$).

7. Chapter 1, problem 9 (a) only).

8. Chapter 1, problem 10.

9. A bag has three balls in it: 1 red, 1 blue and 1 green. Here's an experiment: pick a ball at random, note its color, return it to the bag, then again pick a ball at random, and note its color.
   (a) Write down a sample space for this experiment.
   (b) Calculate the probability that the two colors observed are the same.

10. A system has 5 components, each of which is either working or failed. An experiment consists of observing the 5 components, and recording the status of each one. Record an outcome of this experiment by a vector $(x_1, x_2, x_3, x_4, x_5)$, with $x_i = 0$ if component i is failed, and $x_i = 1$ if it is working.
    (a) How big is the sample space?
    (b) How many outcomes in the event "Component 1 is working, and component 4 is failed"?
    (c) Suppose that the system works if EITHER components 1 and 2 are both working OR components 3 and 4 are both working OR components 1, 3 and 5 are working. Write down all the outcomes in the event "The system is working".

11. A and B are mutually exclusive events with $\Pr(A) = .3$ and $\Pr(B) = .5$.
    (a) What is the probability that either A or B occurs?
    (b) What is the probability that A occurs but B does not?
    (c) What is the probability that both A and B occur?

12. Put a rook at a random square on an 8 by 8 chessboard (all squares equally likely). Put a second rook down on a random different square (all free squares equally likely); keep going, always putting a new rook down on a randomly chosen square that doesn't already have a rook on it (all free squares equally likely). After having put down 8 rooks, what is the probability that no two rooks are attacking each other (that is, on the same row of same column as each other)?

13. There are 5 hotels in downtown South Bend. Last night, 3 travellers arrived from distant La Porte, Indiana. Each of the three picked a hotel at random from among the five to check-in to (each hotel equally likely). The three travellers didn't communicate with each other about their check-in plans.

    (a) What is the probability that all three stayed at different hotels?

    (b) What is the probability that all three stayed at the same hotel?

14. Let E, F and G be three events. Using $\cup$, $\cap$ and ^{c}, find expressions (such as $F \cap G^c$) for each of the following events:

    (a) Only E occurs.

    (b) At least one of E, F and G occurs.

    (c) All three of E, F and G occur.

    (d) At most one of E, F and G occur.

    (e) Exactly two of E, F and G occur.

15. Use the axioms of probability to show that if E and F are any events, then

$$\Pr(E \cap F) \geq \Pr(E) + \Pr(F) - 1.$$

16. Use a Venn diagram to explain why the following relations between events are always true:

    (a) $E \cap F \subseteq E \subseteq E \cup F$.

    (b) If $E \subseteq F$ then $F^c \subseteq E^c$.

17. Go to Random.org's dice roller (`http://www.random.org/dice/`), and select two dice. Roll the two dice 50 times, and each time record the difference between the higher and the lower number rolled (it will be one of 0, 1, 2, 3, 4 or 5). Record your results in a table like the one below (your actual "times rolled" numbers may be different):

difference	0	1	2	3	4	5
times rolled	3	14	15	8	6	4

    (a) Let p_i be the probability that when you roll two dice, the difference between the two numbers you roll is exactly i. Use your data to empirically guess at the values of p_0, p_1, p_2, p_3, p_4 and p_5.

    (b) Use your guesses to estimate the probability that when you roll two dice, the difference between the two numbers you roll is at least 3.

    (c) By listing all the events in the sample space of roll two dice, calculate p_i exactly for each i, and calculate the exact probability that the difference between the two numbers is at least 3.