Introduction to Probability, Fall 2013

Math 30530 Section 01

Homework 10 — due in class Monday, December 9

General information

At the top of the first page, write your name, the course number and the assignment number. If you use more than one page, you should **staple all your pages together**. The grader reserves the right to leave ungraded any assignment that is disorganized, untidy or incoherent.

Reading

- Sections 4.1, 4.4 and 4.2

Problems

1. (a) Let X be a uniformly selected random number on the interval $[0, 1]$. For $a > 0$ and $b \in \mathbb{R}$, let $Y = aX + b$. Calculate the density function of Y.

   (b) Write down the density function of a uniformly selected random number on the interval $[b, a + b]$ ($a > 0$ and $b \in \mathbb{R}$).

   Hint: your answer to both parts should be the same — if $X \sim \text{Uniform}(0, 1)$, then $aX + b \sim \text{Uniform}(b, a + b)$.

2. I throw a dart n times at a dartboard with radius 1, each time selecting a uniform and independent point from the board. Let X_i be the random variable that records the distance from my ith throw to the center of the dartboard, and let $Y_{(n)}$ be the distance to the center of the dartboard of my *closest* throw (i.e.

$$Y_{(n)} = \min\{X_1, \ldots, X_n\}).$$

   (a) Find the density function of $Y_{(n)}$.

   (b) For $n = 1, 2, 3, 4$, find $E(Y_{(n)})$.

   (c) For $n = 1, 2, 3, 4$, find $\Pr(Y_{(n)} < .5)$.

   Note: we did $n = 2$ in class.

3. Use the transform of the exponential random variable (which we calculated in class) to compute $E(X)$ and $\text{Var}(X)$ when $X \sim \text{exponential}(\lambda)$.

4. (a) Let $X \sim \text{Poisson}(\lambda)$. Calculate the transform X.

   (b) Let $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$ be independent. Use transforms to show that $X + Y \sim \text{Poisson}(\lambda_1 + \lambda_2)$.

5. I choose r items from a collection of $N + M$ items, one after the other, *without replacement*. N of the items are "good" and the remaining M are "bad". Let X_i be the indicator random variable indicating whether the ith item I chose was good (so $X_i = 1$ if the ith item was good, and $X_i = 0$ if it was bad). For $i \neq j$, calculate the covariance $\text{Cov}(X_i, X_j)$, and the correlation coefficient. (**Note**: it should be very small, going to 0 as N and M go to infinity; this justifies treating samples without replacement as being essentially independent when the population is large).

6. Chapter 4, problem 29.

7. Chapter 4, problem 30.

8. Chapter 4, problem 17.

9. Chapter 4, problem 18.

10. Chapter 4, problem 19.