Introduction to Probability, Fall 2013

Math 30530 Section 01

Homework 6 — due in class Friday, October 11

General information

At the top of the first page, write your name, the course number and the assignment number. If you use more than one page, you should **staple all your pages together**. The grader reserves the right to leave ungraded any assignment that is disorganized, untidy or incoherent.

Reading

- Sections 2.4 and 2.5

Problems

1. Let X be a Poisson random variable with parameter λ. Show that the variance of X is λ.

2. I roll two dice. Alice looks at the maximum of the two numbers that come up, and calls this X. Bob looks at the minimum, and calls it Y (for example, if the roll leads to a 3 and a 5, then $X = 5$ and $Y = 3$; if the roll leads to two 6's, then $X = Y = 6$).

   (a) Write down the joint mass function of X and Y in a 6 by 6 table.

   (b) After the roll, Bob pays Alice $X^2 - Y^2$ dollars. Calculate the expected number of dollars that Alice receives.

3. Four students, 1, 2, 3 and 4, take a make-up quiz, which is made up of 5 parts, i,ii,iii,iv and v. 1 answers only parts i,ii and iii. 2 and 3 both answer only parts i, iv and v. 4 answers all 5 parts. This means that between the four students, there are 14 question-parts completed. I pick one of these 14 at random to grade first, and I record the following pair of numbers: X, which is the number of the student whose quiz I have chosen, and Y, which is the part number of the answer I am about to grade (I record $Y = 1$ if it is part i, $Y = 2$ if it is part ii, etc.).

   (a) Write down the joint mass function of X and Y in a table.

   (b) Find the marginal mass function of X **using the table**.

   (c) Find the marginal mass function of Y **using the table**.

4. Chapter 2, problem 24

5. Chapter 2, problem 26

6. Back to Alice and Bob: Alice repeatedly rolls a dice until the first time that she rolls a 6, and lets X be the number of attempts it took her. Independently, Bob repeatedly rolls a pair of dice until the first time that he sees a pair of 6's, and lets X be the number of attempts it took her.

   (a) Which pairs (x, y) are such that there is a non-zero probability that $X = x$ and simultaneously $Y = y$?

   (b) For each such pair (x, y), calculate $\Pr(X = x, Y = y)$ (i.e., the value of the joint mass function $p_{X,Y}(x, y)$).

   (c) Use the joint mass function to calculate the probability that Alice and Bob finish their experiments after the same number of trials (that is, that $X = Y$).

   (d) Use the joint mass function to calculate the probability that Alice finishes her experiment before Bob does (that is, that $X < Y$).