

# Introduction to Probability, Fall 2013

## Math 30530 Section 01

### Homework 7 — Solutions

1. Chapter 2, problems 38, 39, 40 and 41 (a, b, c only) — see Homework 7 solutions file 1 on website.
2. By writing the Negative Binomial random variable with parameters  $p$  (success probability on each trial) and  $m$  (number of successes needed until experiment stops) as a sum of independent geometric random variables, calculate the mean and variance (in terms of  $p$  and  $m$ ).

**Solution:** Let  $X \sim \text{NegativeBinomial}(m, p)$ . We can write  $X = X_1 + X_2 + \dots + X_m$ , where  $X_1$  is the number of trials until the first success,  $X_2$  is the number of trials after the first success until the second success, and so on. Each  $X_i$  is a geometric random variable with parameter  $p$ , and because the individual trials are independent, the  $X_i$ 's are independent. So we can use our knowledge of the mean and variance of the geometric to calculate both the mean and variance of the negative binomial:

$$E(X) = E(X_1 + \dots + X_m) = E(X_1) + \dots + E(X_m) = \frac{m}{p}$$

and

$$\text{Var}(X) = \text{Var}(X_1 + \dots + X_m) = \text{Var}(X_1) + \dots + \text{Var}(X_m) = \frac{m(1-p)}{p^2}.$$

The expectation calculation would have worked, whether or not the  $X_i$ 's were independent; the variance calculation only works because of independence.

3. Chapter 3, problems 1, 2, 5, 6, 7 and 8 — — see Homework 7 solutions file 2 on website.