Introduction to Probability, Fall 2013

Math 30530 Section 01

Homework 8 — due in class Friday, November 15

General information

At the top of the first page, write your name, the course number and the assignment number. If you use more than one page, you should **staple all your pages together**. The grader reserves the right to leave ungraded any assignment that is disorganized, untidy or incoherent.

Reading

- Sections 3.3 and 3.4

Problems

1. Verify that the variance of the standard normal random variable Z with parameters $\mu = 0$ and $\sigma^2 = 1$ is indeed 1, by explicitly computing $\int_{-\infty}^{\infty} (1/\sqrt{2\pi}) x^2 e^{-x^2/2} dx$. You can assume that $E(Z) = 0$ and that the density function for Z, $f(x) = (1/\sqrt{2\pi}) e^{-x^2/2}$, is indeed a density function, that is that $\int_{-\infty}^{\infty} (1/\sqrt{2\pi}) e^{-x^2/2} dx = 1$.

2. Chapter 3, problem 11

3. Chapter 3, problem 12

4. Chapter 3, problem 13

5. The temperature of a steel rod four hours after tempering is known to be normally distributed with mean 75 degree Celsius and standard deviation 25.

   (a) Compute the probability that a rod has temperature above 105 degrees Celsius four hours after tempering.

   (b) I can begin using a rod after tempering once its temperature has dropped below 105 degrees Celsius. If 10 rods are set aside for four hours after tempering, whats the probability that I can use at least 7 of them at this time?

6. A soft drink machine can be regulated so that it discharges an average of μ ounces per cup. If the amount the machine dispenses is modeled by a normal random variable with standard deviation 0.3 ounces, find a value of μ such that 8-ounce cups will over flow only 1% of the time.

7. The distribution of resistance for resistors of a certain type is known to be normal. 9.85% of all resistors have a resistance exceeding 10.257 Ohms, and 5.05% have resistance smaller than 9.671 Ohms. What are the mean value and standard deviation of the resistance distribution?

8. Chapter 3, problem 15

9. My dog Casey has run off into the forest. Painful past experience has taught me that the time until Casey is sprayed by a skunk is exponentially distributed with an average (expected value) of 2 hours. If the time it takes me to run back home and return with Casey's favorite squeak-toy ranges between 20 and 40 minutes, and is uniformly distributed over that interval, Calculate the probability that I succeed in luring Casey back with her favorite toy before she gets sprayed by a skunk. (Assume that Casey comes back to me the moment I return with the squeak-toy).

10. An introverted professor X rarely turns her face away from the blackboard. The moment when she first faces her students is equally likely to occur at any point during her hour-long lecture. X's student, Y, is very busy. He's always at least 10 minutes late, though he always manages to get to class (at a completely random moment) before the lecture is halfway through. How likely is it that when X faces the class for the first time, she'll see Y eagerly taking notes?

11. The joint density of a pair of random variables X, Y is

$$f(x,y) = \begin{cases} Cxe^{-4y} & \text{if } 0 \leq x \leq 4 \text{ and } 0 \leq y \leq \infty \\ 0 & \text{otherwise.} \end{cases}$$

    (a) Find C.
    (b) Are X and Y independent? Explain.
    (c) Calculate $\Pr(X + Y \geq 4)$.
    (d) Find the marginal density of X.

12. Let X and Y be independent uniform random variables, both taking values between -1 and 1. Find the probability that the quadratic equation

$$t^2 - Xt + Y = 0$$

has real roots.