# Math 30530 — Introduction to Probability

Quiz 1 – Wednesday September 10, 2013

Solutions

1. **Use the Probability Axioms** to show that for any event $A$, $\Pr(A^c) = 1 - \Pr(A)$. Each time you use one of the axioms, say that you are using it.

   **Solution**: $A$ and $A^c$ are disjoint, so by Axiom 2,
   $$\Pr(A \cup A^c) = \Pr(A) + \Pr(A^c).$$
   But also $A \cup A^c = \Omega$, and by Axiom 1, $\Pr(\Omega) = 1$, so also
   $$\Pr(A \cup A^c) = 1.$$
   Combining the two displayed equations we get $\Pr(A) + \Pr(A^c) = 1$ or
   $$\Pr(A^c) = 1 - \Pr(A).$$

2. 20% of students on campus read the *Observer* online each day, and 65% read the paper copy. 20% neither go online to read the *Observer*, nor look at a paper copy. A randomly chosen student is found to be an online reader of the *Observer*. What's the probability that (s)he reads the paper copy?

   **Solution**: Let $P$ be the event that a randomly chosen student reads the paper version, and $O$ the event that a randomly chosen student reads the online version. We want
   $$\Pr(P|O) = \frac{\Pr(P \cap O)}{\Pr(O)}.$$
   We are given $\Pr(O) = .2$. To calculate $\Pr(P \cap O)$ we use $\Pr(P \cup O) = \Pr(P) + \Pr(O) - \Pr(P \cap O)$, so $\Pr(P \cap O) = \Pr(P) + \Pr(O) - \Pr(P \cup O)$. We know $\Pr(O) = .2$ and $\Pr(P) = .65$, and also $\Pr((O \cup P)^c) = .2$, so $\Pr(O \cup P) = 1 - .2 = .8$. So $\Pr(P \cap O) = .65 + .2 - .8 = .05$. This lets us say
   $$\Pr(P|O) = \frac{.05}{.2} = .25.$$

3. Use the definition of conditional probability to show that, for any events $A$ and $B$, $\Pr(A^c|B) = 1 - \Pr(A|B)$.

   **Solution**:
   $$\begin{aligned}\Pr(A^c|B) &= \frac{\Pr(A^c \cap B)}{\Pr(B)} \\ &= \frac{\Pr(B) - \Pr(A \cap B)}{\Pr(B)} \\ &= 1 - \frac{\Pr(A \cap B)}{\Pr(B)} \\ &= 1 - \Pr(A|B).\end{aligned}$$