

Math 30530, Probability

Quiz 2, Wednesday February 20

Solutions

1. 2% of all Rayovac AA batteries, and 5% of Duracell AA batteries are defective. One quarter of the AA batteries in my battery drawer are Rayovac (the rest are Duracell). I take an AA battery from my drawer at random, and find it to be defective. How likely is it to have been a Rayovac?

Solution: Let R be the event that the battery chosen is Rayovac, D Duracell (so $R \cup D = S$, the whole sample space, and they are disjoint), and let B be the event that the battery is defective. By Bayes,

$$P(R|B) = \frac{P(B|R)P(R)}{P(B|R)P(R) + P(B|D)P(D)} = \frac{(.02)(0.25)}{(.02)(0.25) + (.05)(0.75)} = 0.1176\dots$$

2. (a) Let A, B, C be three events in a probability space. What is the definition of “the collection of events A, B, C is independent”?

Solution: Each of $P(A \cap B) = P(A)P(B)$, $P(A \cap C) = P(A)P(C)$, $P(B \cap C) = P(B)P(C)$ and $P(A \cap B \cap C) = P(A)P(B)P(C)$ hold.

- (b) I roll three balanced (fair) die — a four-sided one, with faces marked “1” through “4”, a five-sided one, with faces marked “1” through “5”, and a six-sided one, with faces marked “1” through “6”. How likely am I to see the same number on each die?

Solution: The probability that all three show “1” is, by independence of the three die, $(1/4)(1/5)(1/6) = 1/120$. This is also the probability that all three show each of “2”, “3” or “4”. Since these are the only possibilities for the common number, the probability is $4(1/120)$ or $1/30$.