Math 40210: Basic Combinatorics, Fall 2012

Midterm Exam

General information

When is the exam?

In class on Monday, October 8

What does the exam cover?

The following sections from Harris, Hirst and Mossinghoff's *Combinatorics and Graph Theory*, and some slides from the course website:

- Introduction to Chapter 1
- Section 1.1.1
- Section 1.1.2, up to the end of the paragraph that begins on the bottom of page 7
- Section 1.1.3
- Introduction to Section 1.2
- Section 1.2.1 (but not the proof of Theorem 1.6)
- Section 1.2.2, up to the end of the proof of Theorem 1.7
- Section 1.3.1
- Section 1.3.2 (but not the proofs of Theorems 1.15 and 1.16)
- Section 1.3.3
- Section 1.3.4 (not the proof of Theorem 1.9)
- Slides on the course webpage about Prüfer codes
- Intro to Section 1.4
- Section 1.4.1

- Section 1.4.2
- Section 1.4.3, up to the statement of Theorem 1.23
- Slides on **P** versus **NP** from the course website
- Introduction to Section 1.5
- Section 1.5.1
- Section 1.5.2
- Section 1.5.3
- Section 1.5.4 (not the sketch of the proof of Thm 1.40)
- Section 1.6.1
- Section 1.6.2, excluding the proof of Theorem 1.43
- Section 1.6.3

What should I know?

Here's a detailed list of topics that I consider examinable. For the theorems, I expect you to know statement **and** proof, except for the ones marked with a (*), where you should have a good idea of the proof, but you shouldn't expect to have to reproduce the entire proof.

For problems where you have to think, I'll make sure they are similar to homework problems!

- All the basic definitions listed at
  `http://nd.edu/~dgalvin1/40210/40210_F12/40210graphdefinitions.pdf`,
  up to **independence number**.
- Theorem 1.1 (sum of degrees equals twice number of edges)
- Theorem 1.2 (every walk contains a path)
- The special families of graphs listed in Section 1.1.3
- Theorem 1.3 (bipartite $\iff$ no odd cycle)
- Theorem 1.4 (diameter is between radius and twice radius)
- Theorem 1.5 (every graph is center of some graph)
- Theorem 1.7 (powers of adjacency matrix count walks)

- Theorems 1.10,11,12 (a tree is characterized by any two of: $n-1$ edges, connected, acyclic)
- Theorem 1.14 (a tree has at least two leaves)
- Kruskal's algorithm
- Theorem 1.17(*) (Kruskal's algorithm works)
- Cayley's formula
- How to construct a Prüfer code from a tree, and vice versa, and properties of Prüfer codes
- Statement of the matrix tree theorem
- Theorem 1.20(*) (characterization of Euler circuits)
- Corollary 1.21 (characterization of Euler trails)
- Theorem 1.22(*) (Dirac's theorem giving sufficient conditions for a Hamiltonian cycle)
- Theorem 1.23(*) (Ore's theorem giving sufficient conditions for a Hamiltonian cycle)
- Theorem 1.31 (Euler's formula)
- Theorem 1.32 ($K_{3,3}$ not planar)
- Theorem 1.33 (bound on number of edges in connected planar graph)
- Theorem 1.34 (K_5 not planar)
- Theorem 1.35 (planar graphs have a vertex of degree at most 5)
- Statement of Kuratowski's theorem
- Greedy algorithm for coloring
- Theorems 1.41,42,44,45 (various bounds on chromatic number)
- Statement of Brooks' theorem
- Theorem 1.47(*) (5-color theorem)

Notice some things which are missing: polyhedra, Mycielski construction, **P** versus **NP**.

What is the format?

There will be five or six free response questions, similar to homeworks and quizzes.

What can I do to prepare?

- Review the material!
- Review the homeworks and quizzes! Solutions are on the course website.
- Come talk to me! I've tentatively set office hours for the following times:
  - Thursday 3.30-5.30, HH248 (this is a joint office hour with my 30530 class, who also have homework and an exam coming up. I request that if at all possible you come in the **second** hour; I'll be asking my 30530 students to come in the first).
  - Saturday 5-6, room Hayes-Healy 229. Use the door to Hayes-Healy that faces out towards LaFortune; I'll make sure that it's open.
  - Monday from 9.15 to class time. I couldn't find a room for this, so I'll just be sitting in the lobby of the ROTC building.