Math 40210: Basic Combinatorics, Fall 2012

Final Exam

General information

When is the exam?

Monday December 10 at 4.15pm in Pasquerilla 105.

What does the exam cover?

The following sections from Harris, Hirst and Mossinghoff's *Combinatorics and Graph Theory*, and some slides from the course website:

- Introduction to Chapter 1
- Section 1.1.1
- Section 1.1.2, up to the end of the paragraph that begins on the bottom of page 7
- Section 1.1.3
- Introduction to Section 1.2
- Section 1.2.1 (but not the proof of Theorem 1.6)
- Section 1.2.2, up to the end of the proof of Theorem 1.7
- Section 1.3.1
- Section 1.3.2 (but not the proofs of Theorems 1.15 and 1.16)
- Section 1.3.3
- Section 1.3.4 (not the proof of Theorem 1.9)
- Slides on the course webpage about Prüfer codes
- Introduction to Section 1.4
- Section 1.4.1

- Section 1.4.2
- Section 1.4.3, up to the statement of Theorem 1.23
- Slides on **P** versus **NP** from the course website
- Introduction to Section 1.5
- Section 1.5.1
- Section 1.5.2
- Section 1.5.3
- Section 1.5.4 (not the sketch of the proof of Thm 1.40)
- Section 1.6.1
- Section 1.6.2, excluding the proof of Theorem 1.43
- Section 1.6.3
- Introduction to 1.7
- Section 1.7.1
- Section 1.7.2
- Section 1.7.3
- Section 1.7.4, up to (and including) the statement of Theorem 1.59
- Section 1.8.1
- Section 1.8.2
- Introduction to Chapter 2
- Section 2.1
- Section 2.2
- Section 2.3
- Section 2.4 (but not the example about approximating irrationals by rationals)
- Section 2.5
- Section 2.6.2

- Section 2.6.4
- Section 2.6.5
- Section 2.6.6
- Introduction to Section 2.9
- Section 2.9.1

What should I know?

Here's a detailed list of topics that I consider examinable. For the theorems, I expect you to know statement **and** proof, except for the ones marked with a (*), where you should have a good idea of the proof, but you shouldn't expect to have to reproduce the entire proof.

For problems where you have to think, I'll make sure they are similar to homework problems!

- All the basic definitions listed at `http://nd.edu/~dgalvin1/40210/40210_F12/40210graphdefinitions.pdf`.
- Theorem 1.1 (sum of degrees equals twice number of edges)
- Theorem 1.2 (every walk contains a path)
- The special families of graphs listed in Section 1.1.3
- Theorem 1.3 (bipartite $\iff$ no odd cycle)
- Theorem 1.4 (diameter is between radius and twice radius)
- Theorem 1.5 (every graph is center of some graph)
- Theorem 1.7 (powers of adjacency matrix count walks)
- Theorems 1.10,11,12 (a tree is characterized by any two of: $n-1$ edges, connected, acyclic)
- Theorem 1.14 (a tree has at least two leaves)
- Kruskal's algorithm
- Theorem 1.17(*) (Kruskal's algorithm works)
- Cayley's formula
- How to construct a Prüfer code from a tree, and vice versa, and properties of Prüfer codes

- Statement of the matrix tree theorem
- Theorem 1.20(*) (characterization of Euler circuits)
- Corollary 1.21 (characterization of Euler trails)
- Theorem 1.22(*) (Dirac's theorem giving sufficient conditions for a Hamiltonian cycle)
- Theorem 1.23(*) (Ore's theorem giving sufficient conditions for a Hamiltonian cycle)
- Theorem 1.31 (Euler's formula)
- Theorem 1.32 ($K_{3,3}$ not planar)
- Theorem 1.33 (bound on number of edges in connected planar graph)
- Theorem 1.34 (K_5 not planar)
- Theorem 1.35 (planar graphs have a vertex of degree at most 5)
- Statement of Kuratowski's theorem
- Greedy algorithm for coloring
- Theorems 1.41,42,44,45 (various bounds on chromatic number)
- Statement of Brooks' theorem
- Theorem 1.47(*) (5-color theorem)
- Theorem 1.50 (Berge's theorem on maximum matchings)
- Theorem 1.51(*) (Hall's theorem)
- Theorem 1.52 (systems of distinct representatives)
- Theorem 1.53(*) (König-Egerváry theorem)
- Theorem 1.59(*) (Tutte's theorem)
- Definition of the Ramsey number $R(p, q)$
- Sum rule, product rule for counting
- The number of ordered lists, and the number of unordered selections, of k items from n
- Algebraic and combinatorial definitions of binomial coefficients
- Basic binomial coefficient identities:

  - symmetry
  - Pascal's identity
  - binomial theorem
  - summing on upper index
  - Vandermonde convolution

- The multinomial coefficient and the multinomial theorem
- Basic uses properties of the multinomial coefficients
- Counting anagrams of a given length
- The pigeon-hole principle and the Erdős-Szekeres monotone subsequences theorem
- The principle of inclusion-exclusion, and applications:
  - the Euler φ-function
  - derrangements
  - chromatic polynomials

- Selecting m elements from an inexhaustible supply of each of n types of objects, and putting m identical objects into n distinguishable bins
- The method of generating functions to get solutions to linear recurrences
- The Fibonacci numbers
- The Tower of Hanoi problem and generalizations
- The Catalan recurrence and the Catalan numbers
- The generalized binomial theorem
- The Gale-Shapley algorithm for finding a stable matching

Notice some things which are missing: polyhedra, Mycielski construction, **P** versus **NP**, guarding an art gallery.

What is the format?

The exam will consist of free response questions, similar to homeworks, quizzes and the midterm.

What can I do to prepare?

- Review the material!
- Review the homeworks and quizzes! Solutions are on the course website.
- Do a practice exam! Last year's final is available at `http://nd.edu/~dgalvin1/40210/40210_S12/40210S12-Final.pdf`.
- Come talk to me! I've tentatively set office hours for the following times:
  - Friday 1-2, HH248
  - Sunday 4-5, HH248
  - Monday 1.15-2.30, HH248