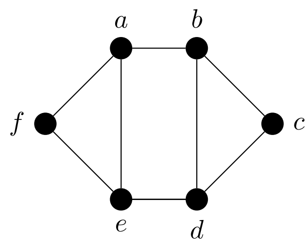
Basic Combinatorics (Math 40210) Sec 01, Fall 2012, Quiz 1

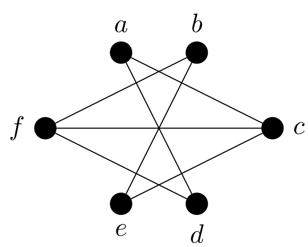
Solutions

1. For the graph G drawn below, write down an a-d walk that is *not* a path.



   Solution: any walk along edges of the graph that starts at a, ends at d, and repeats an edge and/or a vertex will do. For example, $a - e - f - a - e - d$.

2. Draw the complement $\overline{G}$ of G (the graph shown *above*), using the picture below as a starting point. Is $\overline{G}$ bipartite? If it **is**, write down a valid partition $X \cup Y$ of the vertex set. If it **is not**, say why not.



   Solution: $\overline{G}$ is bipartite: setting $X = \{e, e, f\}$ and $Y = \{b, c, d\}$, we have no $X - X$ or $Y - Y$ edges (and all vertices covered, and X and Y disjoint).

3. The *trace* of an n by n matrix A, denoted by $\mathrm{Tr}(A)$, is the sum of the entries down the main diagonal (so for example $\mathrm{Tr}\begin{pmatrix} 2 & 1 \\ 5 & 7 \end{pmatrix} = 9$). Show that for any graph with m edges and with adjacency matrix A, $\mathrm{Tr}(A^2) = 2m$.

   Solution: The ii entry of A^2 counts the number of walks of length 2 that start and end at vertex i; this is exactly $d(i)$, the degree of i (one such walk for each edge). So the trace of A^2 is the sum of the degrees of the graph; by a theorem in class, this is twice the number of edges or $2m$.