

Basic Combinatorics (Math 40210) Sec 01, Fall 2012, Quiz 2

Solutions

September 20, 2012

1. Draw the tree on vertex set $\{1, 2, 3, 4, 5, 6, 7\}$ whose Prüfer code is $(3, 2, 3, 2, 1)$.

Solution (3 pts): 4 and 6 are leaves joined to 3; 3 is joined to 2, which is also joined to 1 and 5. 1 is also joined to a leaf 7. Here is the order in which these edges are added back in in the “undoing” of the Prüfer code: 43, 52, 63, 32, 21, 17.

2. Explain why a tree with maximum degree Δ must have at least Δ leaves

Solution (4 pts): Remove a vertex with degree Δ ; the tree breaks into Δ components. If any of these components are stumps, those vertices were leaves in the original graph. If any component is not a stump, then (by a theorem from class) it is a tree with at least one edge so at least 2 leaves. At least one of these leaves continues to be a leaf when the vertex of degree Δ is put back in (that vertex can increase the degree of one vertex per component). So each of the Δ components contributes at least one leaf to the tree, giving at least Δ leaves in all.

3. Which complete graphs have an Euler circuit?

Solution (3 pts): To have an Euler circuit, all degrees must be even. K_n has all degrees $n - 1$. So $n - 1$ must be even, and n odd. Conclusion: the complete graphs on an odd number of vertices have an Euler circuit.