Basic Combinatorics (Math 40210) Sec 01, Fall 2012, Quiz 3

Solutions

October 5, 2012

1. Define (carefully) $\chi(G)$, the chromatic number of a graph G.

   Solution: The chromatic number of G, $\chi(G)$, is the smallest integer k such there exists a coloring of the vertices of G using k colors, with no two adjacent vertices of G receiving the same color.

2. Explain why $\chi(G) \geq n/\alpha(G)$ for every graph G, where $\chi(G)$ is the chromatic number of G and $\alpha(G)$ is the size of the largest independent set

   Solution: Let K be a coloring using $\chi(G)$ colors. Let C_i be the set of vertices colored i by K. Since C_i is an independent set, we have $|C_i| \leq \alpha(G)$. Summing over all i gives $n = \sum_i |C_i| \leq \chi(G)\alpha(G)$, which is the same as $n/\alpha(G) \leq \chi(G)$.

3. What is the best that can be said about $\chi(C_7)$, the chromatic number of the 7-cycle, if all we know about chromatic number is the bound from the last part of the question?

   Solution: $\alpha(C_7) = 3$ (take every second vertex, as long as possible; clearly there is no independent set of size 4), so we can say $\chi(C_7) \geq 7/3 = 2.33$. Since $\chi(C_7)$ is an integer, we can therefore say $\chi(G) \geq 3$.