Some problems leading to Catalan numbers
The Catalan recurrence, and some values

\begin{align*}
c_0 &= 1 \\
c_n &= c_0 c_{n-1} + c_1 c_{n-2} + \ldots + c_{n-1} c_0 \\
&= \sum_{k=0}^{n-1} c_k c_{n-1-k} \text{ for } n \geq 1
\end{align*}

\begin{align*}
c_0 &= 1 \\
c_1 &= 1 \\
c_2 &= 2 \\
c_3 &= 5 \\
c_4 &= 14 \\
c_5 &= 42
\end{align*}
Some problems leading to Catalan numbers

- **Handshakes**: \( c_n \) counts number of ways that \( 2n \) people in a circle can pair off to shake hands, with no crossing hands.

- **One-sided tied games**: \( c_n \) counts number of ways the Cubs and White Sox can play to an \( n \)-\( n \) tie, in which the Cubs never lead (games considered by the order in which the runs are scored).

- **Triangulations**: \( c_n \) counts the number of different ways that a convex \((n+2)\)-gon can be fully triangulated.

- **Trees**: \( c_n \) counts the number of full binary trees with \( n + 1 \) leaves. (Start with a root. Each vertex either has two children (right and left), or no children.)

- **Tiling stairs**: \( c_n \) counts the number of ways of tiling a height \( n \) staircase with exactly \( n \) rectangles. (The height \( n \) staircase is the set of 1 by 1 boxes whose top right points are the points \((i, j)\) with \( i, j \geq 1 \) and \( i + j \leq n + 1 \).)

- R. Stanley, *Enumerative Combinatorics*, has an exercise that gives 66 different counting problems, all solved by Catalan numbers; an addendum on his website gives 136 more!