SOLUTIONS

(1) I take out two Aces from a standard deck of 52 cards. How many ways are there to select three more cards from the remaining 50, in such a way that the five cards together form a full house (3 of one kind, 2 of another)?

**Solution:** Either my full house consists of 3 Aces and 2 of another card, or 2 Aces and 3 of another card.

In the first case, there are \( \binom{2}{1} \) ways to choose 1 Ace from the two that remain in the deck, then 12 ways to choose the value of the card in the pair, and then finally \( \binom{4}{2} \) ways to choose 2 cards of that value from the possible 4, leading to a total of \( \binom{2}{1} \cdot 12 \cdot \binom{4}{2} \) selections.

In the second case, there are 12 ways to choose the value of the card in the three-of-a-kind, and then \( \binom{4}{3} \) ways to choose 3 cards of that value from the possible 4, leading to a total of \( 12 \cdot \binom{4}{3} \) selections.

In total, the number of selections is

\[
\binom{2}{1} \cdot 12 \cdot \binom{4}{2} + 12 \cdot \binom{4}{3} = 192.
\]

(2) A basic identity involving binomial coefficients is that \( k \binom{n}{k} = n \binom{n-1}{k-1} \). Give a combinatorial proof of this identity, by showing that both sides of the identity count the same thing (in different ways).

**Solution:** One way to select a committee of \( k \) people from a group of \( n \) people, with one person on the committee being designated the chair, is to first choose the committee (\( \binom{n}{k} \) choices) and then choose the chair from among the committee members (\( k \) choices), leading to a total of \( k \binom{n}{k} \) choices.

Another way is to first choose the chair (\( n \) choices) and then choose the remaining \( k - 1 \) members of the committee from the remaining \( n - 1 \) people (\( \binom{n-1}{k-1} \) choices), leading to a total of \( n \binom{n-1}{k-1} \) choices.

Since both methods count the same thing, we conclude \( k \binom{n}{k} = n \binom{n-1}{k-1} \).

---

Date: Wednesday, March 21.