Suppose you are to design a PI controller for the first-order plant depicted in Fig. 1a. You are to design the controller so that the closed-loop poles lie within the shaded region shown in Fig. 1b.

1) What range of $\omega_n$ and $\zeta$ values correspond to the shaded regions of Fig. 1b? (A simple estimate of the range of values from the figure is sufficient).

2) The closed-loop transfer function will have a zero that will affect the time-domain response. Given your $\omega_n$ and $\zeta$ ranges from 1), and assuming that the zero has a small effect, calculate the corresponding rise time and overshoot ranges of the system.

3) Let $K_a = \alpha = 2$. Find the range of values for $K$ and $K_I$ so that the poles of the closed-loop system lie within the shaded regions.

4) Prove that no matter what the values of $K_a$ and $\alpha$ are, the controller provides enough flexibility to place the poles anywhere in the left-half plane.
Fig. 1. Feedback control system and desired closed-loop pole locations for Problem 3

- **Problem 4**: Plants with right-half plane (RHP) poles (unstable plants) can be stabilized using feedback control; feedback can move the RHP poles to the left-half plane (LHP). Plants with RHP zeros, while stable, have other undesirable properties. Demonstrate and explain why all feedback controllers are unable to move RHP zeros to the LHP.