• **Problem 1:** Last week you analyzed the control law design of an engineer attempting to embed logic within three robot football players that maintains a safe distance between each player. After the engineer’s original failure, (s)he re-wrote the control rules as the following system of differential equations:

\[
\begin{align*}
\dot{\zeta}_1 &= \alpha ((\zeta_2 - \zeta_1) - d) + \alpha ((\zeta_3 - \zeta_1) - 2d) \\
\dot{\zeta}_2 &= -\alpha ((\zeta_2 - \zeta_1) - d) + \alpha ((\zeta_3 - \zeta_2) - d) \\
\dot{\zeta}_3 &= -\alpha ((\zeta_3 - \zeta_1) - 2d) - \alpha ((\zeta_3 - \zeta_2) - d).
\end{align*}
\]  

(1)

Conceptually, this formulation is equivalent to specifying that each robot is virtually massless, has a virtual spring connecting it to the other robots, and has a virtual damper connected to the earth to damp out vibrations (Fig. 1). The unstretched spring length of the short springs is \(d\) and the unstretched length of the long spring is \(2d\). Evaluate whether or not this new scheme achieves the engineering goals.

1) Write the above rules of motion for the 3-robot system in the form \(\dot{\zeta} = A\zeta +Bg(t)\) for \(\alpha = \frac{1}{2}\) and \(d = 1\).

2) Suppose the robots have an initial condition for their positions specified by the vector \(\zeta(0) = [1, 2, 4]^T\). Calculate an expression for \(\zeta_i(t)\) for each robot as \(t\) goes to infinity.

3) Describe your physical interpretation of the movement of the robots. Test and plot the response for different initial conditions to gain an understanding. Does this scheme avoid collisions for all initial conditions?
• **Problem 2:** Consider the eigenvalues of the set of $4 \times 3 \times 3$ real-valued $A$ matrices in Fig. 2. Fig. 3 gives the time-domain plots for the indices of a set of state transitions matrices, $e^{At}$, for the given $A$ matrices (the order of presentation has been scrambled). Match the eigenvalue maps to the state transition matrices. Give a complete justification for each match to receive full credit. You do not need to perform any calculations to receive full credit.

• **Problem 3:** 8.1.4

• **Bonus [5pts]:** This bonus problem is a continuation of Problem 1. Beyond maintaining a safe distance between robots, it is also possible to drive the configuration of robots in time. For instance, one robot can be designated the leader and a virtual force can be applied to this robot (Fig. 4). The other robots will follow through the configuration of virtual springs and dampers.

1) Write the non-homogenous system of 1$^{st}$ order differential equations that describes the configuration of virtual springs, dampers, and forces in Fig. 4.

2) Solve for and plot the trajectories $\zeta_1$, $\zeta_2$, and $\zeta_3$ when $k = 0.5$, $b = 1$, and $F(t) = \cos t$. 

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Fig. 1. Conceptual structure of a control law to specify positions of three robot football players.
Fig. 2. Map of eigenvalue coordinates for 4 different $3 \times 3$ A matrices.
Fig. 3. Time domain solutions of indices of the state transition matrix.
Fig. 4. Identical structure as Fig. 1, but with Robot 2 designated as the leader.