Read: Sections 9.4 – 9.5.

Problems:

• **Problem 1:** Problem 9.8 of Goodwine. Plot the responses when verifying. The Matlab commands tf and step will be useful.

• **Problem 2:** This second problem is designed to get you familiar with PID control and using the Matlab package Simulink to simulate the response of interconnected systems. In this problem, you will investigate the motor position control problem from Homework 6. In homework 6 you had a motor with the transfer function

\[
\frac{\Theta(s)}{I_a(s)} = \frac{k_t}{s(Js + b)}
\]

which is your plant transfer function: \( J = 2, b = 4, k_t = 5 \). You previously investigated the closed-loop system with P-control with \( k_p = 10 \). Now you will test different controller designs, first in theory and then later in simulation.

1) Demonstrate the following.

(a) P-control. \( k_p > 0, k_i = k_d = 0 \). Demonstrate using the theory from the course that the closed-loop system is second order. Find the range of \( k_p \) such that is \( Y(s)/R(s) \) is underdamped and range where \( Y(s)/R(s) \) is overdamped. Also demonstrate that \( y_{ss} = A \) for a \( R(s) = A/s \): note that, with respect to \( y_{ss} \), this mass-damper behavior is different from the mass-spring-damper plant in class.

(b) PD-Control. \( k_p > 0, k_d > 0, k_i = 0 \). Demonstrate using the theory from the course that the closed-loop system is second order. Describe why you would expect the overshoot in the output as a response to a step reference to decrease.
(c) PI-Control. \( k_p > 0, k_i > 0, k_d = 0 \). Demonstrate using theory from the course that the closed-loop system is third order.

2) A Simulink diagram has been set up for you: download Problem_1.slx and open it in Matlab. This simulation investigates a situation where the reference position for the motor instantaneously changes from 0 to 2 rad. Investigate each block (double-click on it) and the tunable parameters; some parameters may not make complete sense yet, but you are just getting familiar with it. Next click the green play button to run the simulation; the simulation will generate data vectors which you can plot for time, reference, input, and actual velocity. Next you will manipulate controller gains by changing the values of P, I, and D gains; ignore the Filter coefficient parameters as the reason for this parameter is beyond the scope of this course.

(a) Test three different P-controllers: one for an overdamped response and two for an underdamped response. Plot the three different responses and the reference all superimposed on the same graph. Comment on whether the response matches your analysis in Part 1a and also the effect of P-control discussed in class.

(b) Choose your favorite P-control design and add an integral term. Make \( k_i \sim \frac{1}{2} k_p \). Plot your P-control response, PI-control response, and the reference all superimposed on the same graph. Comment on whether the response matches the effect of PI-control discussed in class. Note that \( y_{ss} = A \) for the nominal P-Control system.

(c) Use your PI-Control design and now make it PID. Manipulate the \( k_p, k_i, \) and \( k_d \) gains until you get a step response with rise time less than 1 sec and an overshoot less than 10\%. You do not need to perform any handwritten work here, just manipulate the gains to get a feel of how each parameter modifies your response. Plot the output superimposed on the reference signal.