Read: Section 9.6.

Problems:

- **Problem 1:** Problem 9.9 of Goodwine.

- **Problem 2:** Attached with this homework assignment are data sheets for the major components of a feedback control loop for a robotic positioning system. Draw a generic control system feedback loop and show where each physical component belongs in the feedback loop. Give a brief rationale why each component belongs at each designated position.

  Notes: When we analyze controlled systems we draw them as generic block diagrams. This is an abstraction and it makes it appear as though each block is basically the same. In reality, the hardware that each block represents is vastly different. We discussed some hardware components in Lecture 25; your notes will help here. Also, think about the following questions to help answer this problem.

  - What is the purpose of a robotic positioning system? This dictates what the output signal will be.
  - What is the input and output of each element with a spec sheet? What are the units of the input and output signals of each element?
  - For all intents and purposes, a computer can perform any numerical differential equations operation you wish. But, how would you interface a computer with an external piece of hardware?

- **Problem 3:** This problem is the first step of the pendulum project. It is purely theoretical; you do not need to perform experiments at Stinson-Remick.

  1) Derive the equations of motion of a hanging pendulum in terms of parameters $\theta(t)$, $m$, $l$, $b$, $g$, $k_r$, and $i(t)$ (Fig. 1).
Fig. 1. **Pendulum in the hanging configuration.** Motion is also resisted by a bearing torque of $b\dot{\theta}(t)$.

2) Linearize this system about the position $\theta = 0$. Derive the transfer function for the pendulum, $P_1(s) = \frac{\Theta(s)}{I(s)}$, from these linearized equations of motion. Write your transfer function in the form of a second order underdamped system with general parameters $\bar{k}$, $\omega_n$, and $\zeta$.

3) Derive the time-domain response of the linearized system to a pulse of motor current using your transfer function; select a pulse of magnitude $A$ with a pulse period of $T$ sec (Fig. 2). You will be using this equation to identify unknown physical parameters, $\bar{k}$, $\omega_n$, and $\zeta$ in your experimental pendulum system.
4) Now, derive the equations of motion of an inverted pendulum in terms of parameters \( \theta(t), m, l, b, g, k_r, \) and \( i(t) \) (Fig. 3).

5) Linearize this inverted system about the position \( \theta = 0 \). Write the transfer function of your plant, \( P_2(s) = \frac{\theta(s)}{I(s)} \), for this inverted pendulum. Your transfer function will be different for the hanging and inverted configurations.