AME 30315; Spring 2015; Midterm 1 Review (not graded)

Problems:

• 6.1.6; also, find the solution \( \zeta(t) \) for \( \zeta_0 = [\zeta_1(0), \zeta_2(0)]^T \).
• 6.4.6; also, find the solution \( \zeta(t) \) for \( \zeta_0 = [\zeta_1(0), \zeta_2(0)]^T \).
• 6.7.3; also, find the solution \( \zeta(t) \) for \( \zeta_0 = [\zeta_1(0), \zeta_2(0)]^T \).
• 6.10
• 6.17
• 7.3.1 and 7.3.2
• 8.3.4 and 8.3.11
• 8.6.2 and 8.6.4
• 8.11
• 8.13
• 8.16
• 8.21

Consider the circuit in Fig. 1. The system of first-order differential equations that describes the current through the inductor, \( I(t) \), and voltage drop across the capacitor, \( V(t) \), is given by:

\[
\frac{d}{dt} \begin{bmatrix} I(t) \\ V(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I(t) \\ V(t) \end{bmatrix}.
\] (1)

1) Show that the eigenvalues of the coefficient matrix are real and unique if \( L > 4R^2C \) and complex conjugate if \( L < 4R^2C \). If you were designing this circuit, could you tell if the system will oscillate based solely on the values of the resistor, inductor, and
capacitor chosen? Is there a value of $R > 0$, $L > 0$, and $C > 0$ that you could choose such that either $V(t)$ or $I(t)$ blows up to infinity as time goes to infinity?

2) Suppose that $R = 1\Omega$, $C = 1F$, and $L = 1H$. Find the general solution of the electrical circuit.

3) Find $I(t)$ and $V(t)$ if $I(0) = 2A$ and $V(0) = 1V$.

![LRC Circuit](Boyce, 2009)

Fig. 1. **LRC Circuit**

- Use partial fraction expansion to compute $x(t)$ when:

$$X(s) = \frac{4}{s^2 + 2s + 4} \left( \frac{1}{s} \right).$$

Use partial fraction expansion to compute $x(t)$ when:

$$X(s) = \frac{4}{(s^2 + 2s + 4) \left( \frac{s}{20} + 1 \right)} \left( \frac{1}{s} \right).$$

Are the responses similar? Explain whether this was expected or unexpected.

- Match the poles of a transfer function $G(s)$ to the correct impulse response $g(t)$ in Fig. 2. Give you rationale for each match. *Note:* you should not need to do any math to do this.
\( G(s) \) pole locations:

1) \( s = -2, -2 \)
2) \( s = +\sqrt{2}i, -\sqrt{2}i \)
3) \( s = \frac{1}{2} + 4i, \frac{1}{2} - 4i \)
4) \( s = +5i, -5i \)
5) \( s = -2 \)
6) \( s = -\frac{1}{2} + 4i, -\frac{1}{2} - 4i \)

Fig. 2. Various impulse responses

- System modeling problem
Consider the system illustrated in Figure 2.

![System Diagram]

**Figure 2.** System for Problem 3.

In homework 7 you determined that the equations of motion for the system illustrated in Figure 2 were

\[\begin{align*}
2m\ddot{x} + m l \left(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta\right) + kx &= 0 \\
ml^2 \ddot{\theta} + ml \ddot{x} \cos \theta + mgl \sin \theta &= \tau.
\end{align*}\]

If $\theta$ and $\dot{\theta}$ are small, then an approximation to the equations is

\[\begin{align*}
2m\ddot{x} + ml \ddot{\theta} + kx &= 0 \\
ml^2 \ddot{\theta} + ml \ddot{x} + mgl \theta &= \tau. \tag{1} \tag{2}
\end{align*}\]

(a) Using equations 1 and 2 determine the transfer function from the input torque, $\tau(t)$ to the position of the mass, $x(t)$.

![Motor Circuit Diagram]

**Figure 3.** Motor circuit for Problem 3.

(b) If the torque, $\tau$ is applied by a d.c. motor driven by the circuit illustrated in Figure 3, determine the transfer function from the applied voltage, $v(t)$ to the position of the mass, $x(t)$.

• (For those who want a challenge) Multiple Input Problem
Problem 1

Consider a model of a two tank system containing a heated liquid, where \( T_B \) is the temperature of the fluid flowing into the first tank and \( T_J \) is the temperature of the liquid flowing out of the second tank. We would like the temperature of the 2\textsuperscript{nd} tanks to take on some desired value of \( T_{2d} \).

Two tank system schematic

This system of two tanks has a controllable heat input, \( Q \), into the first tank. That is, a controllable amount of heat \( Q \) can be added to modify the temperature of tank 1 and then, by flow exchange, the temperature of tank 2. The block diagram of the temperature control system is given below in the figure.

Block diagram of two tank temperature control system

The time constants are \( \tau_1 = 10 \) seconds and \( \tau_2 = 50 \) seconds.

(a) Determine the temperature \( T_J(s) \) in terms of \( T_B(s) \) and \( T_{2d}(s) \). I'm looking for one equation here: \( T_J(s) = ??? \).

(b) If \( T_{2d}(s) \), the desired output temperature from tank 2, is instantaneously doubled from \( T_{2d}(s) = 1/s \) to \( T_{2d}(s) = 2/s \), where \( T_B(s) = 1/s \), determine the transient time domain response of \( T_J(t) \) when \( G_c = K = 500 \).

(c) Find the steady state error \( e_s \) for the system of part b, where \( E(s) = T_{2d}(s) - T_B(s) \).