**Problem 1:**

1) Problem 4.6 of Meirovitch using superposition and convolution.

2) Without re-doing the convolution integral, what is the system output if the input is instead $F(t) + F(t - \tau)$?

**Problem 2:** Microelectromechanical system (MEMS) oscillators are mass-spring-damper systems with both nonlinear springs and dampers (Fig. 1). A common differential equation to describe this motion is a specific form of the Duffing equation,

$$m\ddot{x} + \left( b_1 + b_2x^2 \right) \dot{x} + k_1x + k_3x^3 = \bar{F} \left( 1 + \cos \omega t \right).$$

Use $m = 1 \times 10^{-9}$ kg, $b_1 = 1 \times 10^{-6}$ N s m$^{-1}$, $b_2 = 1 \times 10^{-3}$ N s m$^{-3}$, $k_1 = 10$ N m$^{-1}$, $k_3 = 1 \times 10^{12}$ N m$^{-3}$, $\bar{F} = 5 \times 10^{-5}$ N, and $\omega = 1 \times 10^5$ rad s$^{-1}$.

1) Linearize this nonlinear ODE about its time averaged mean position (i.e. the static equilibrium when the non-homogeneous forcing function is $\bar{F}$). You can use a computer to calculate potential equilibrium solutions. You will get one real solution and two complex solutions; the complex solutions are not physically realizable and should not be used.

2) Solve the steady-state solution of the linear ODE. Use your favorite method.

3) Plot a few periods of the solution. Be sure to carefully select your axes.

**Problem 3:** Demonstrate that the unit step function is the time-integral of the unit impulse (Dirac delta) function. Demonstrate that the unit ramp function is the time-integral of the unit step function. Analogous relationships hold for the Kronecker delta function, unit step,
and unit ramp functions in discrete-time. Describe how you could compute the unit ramp function knowing the Kronecker delta function in discrete-time.