Problem 1. [easy] Describe in detail the relative merits of control design using Nyquist methods, Bode analysis, and *ad hoc* simulation. Are any of these complementary? Do any of these methods give you information and tools that the others don’t?

Problem 2. [easy] Suppose you’re given the Bode plots for each of the following uncompensated systems. Point out what the problems are likely to be with these systems, and what you would like to do as a control designer to improve performance. Assume the goals are the “typical” things such as good phase and gain margin, good noise rejection, low error to reference inputs, etc.

![Bode Diagram](image)

Figure 1: Part (a)
Figure 2: Part (b)

Figure 3: Part (c)
Problem 3. [easy] Derive a description for the sampling error of a sinusoid with frequency $f_s$ sampled at frequency $f_s$. How does the sampling error change as $f_s \to \infty$? Based on your results, what do you think a good relationship would be between $f$ and $f_s$? For the last question, speak in terms of a theoretical limit on $f_s$ and a practical limit for $f_s$, including some practical considerations.

Problem 4. [easy] Explain the theory behind the Nyquist stability criteria?

Problem 5. [easy] Consider the unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{K}{s(s^2 + s + 4)}.$$ 

Determine the value of $K$ such that the phase margin is $50^\circ$. What is the gain margin with this gain $K$?

Problem 6. [easy]

Consider the control system shown in Figure 4. Determine the gain $K$ and time constant $T$ of the controller $G_c(s)$ such that the closed-loop poles are located at $s = -2 \pm 2i$.

$$\begin{array}{c}
\text{Figure 4: Closed-loop control scheme.}
\end{array}$$

Problem 7. [hard] Consider the closed-loop system whose open-loop transfer function is

$$G(s) = \frac{Ke^{-2s}}{s}.$$ 

Determine the maximum value of $K$ for which the system is stable.

Problem 8. [easy] Given

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$ (1) 

show that

$$|G(j\omega_n)| = \frac{1}{2\zeta} \text{ and } \angle G(j\omega_n) = -\pi/2.$$ (2)

Problem 9. [hard] A well-known approximation for the damping ratio in terms of the phase margin is $\zeta \approx \frac{PM}{100}$. State the assumptions that go into this approximation and then derive the approximate relationship.

Problem 10. [easy] Demonstrate that a lead-lag controller can be designed to approximate a PID controller. If this PID controller has gains $k_p$, $k_i$, and $k_d$, write the transfer function of the approximating lead-lag controller.

Problem 11. [hard] In Homework 6 you designed a feedback controller for joint 1 of a serial positioning robot. This feedback controller was designed given an average model of the transfer function for joint 1 (black line in Figs. 8 and 9 of Bukkems paper). Also given
in Fig. 8 of Bukkems is the experimental frequency response when tested at 16 different static postures for joints 2 and 3 (gray lines in Fig. 8, see Fig. 3 for diagram of all the joints). Now consider the design of a feedforward controller to be designed as an add-on to your existing feedback controller. What complications should you consider when implementing a feedforward controller based on this average model? *Note:* I’m not saying that a feedforward controller would not be effective for this system, it would probably help performance some, but there are some potential complications that should be considered before adding a feedforward controller.


**Problem 13.** [hard] Demonstrate and/or explain why the following rule of thumb is advisable: if you have a plant with no poles or zeros in the right half plane the slope of your magnitude bode plot should be approximately -20 db/dec at the crossover frequency.

**Problem 14.** [easy] Draw the open-loop transfer function, \( K(s)P(s) \), exclusion regions on a typical bode plot graph paper if you want your system to track reference signal frequency components up to 10 rad/sec with 5% attenuation and have 95% sensor noise rejection of noise signals with a frequency of 150 rad/sec or greater.

**Problem 15.** [easy] What do you want the DC gain of your typical closed loop transfer function to be, and why? Describe the idea of closed-loop bandwidth?