AME 30315; Spring 2013; Final Exam Review (not graded)

Problems:

- 8.3.12
- 8.3.15
- 8.6.5
- 8.9
- 9.20
- 9.21
- 9.38
- 10.10.1–3
- 10.14.3–4
- 10.15.2–3
- 10.18.2, 10.18.4 (change $\phi_m$ by $30^\circ$ instead).
- 10.19 (assume that the zero of the lead controller and PD controller is at 2 rad/sec and that the pole of the lead controller is at 20 rad/sec).
- 2010 AME30315 Third Exam (off Goodwine’s website) is a good midterm to review.
- Consider the type 1 system in Fig. 1. We would like to design the controller $K(s)$ to meet the following requirements: 1) The steady state change of $y(t)$ due to a constant unit disturbance $w(t) = 1(t)$ should be less than $\frac{4}{5}$, and 2) the damping ratio $\zeta = 0.7$. Using the root-locus techniques:
  
  (a) Show that proportional control is not adequate.
  
  (b) Show that proportional derivative control will work.
  
  (c) Find values of the gains $k_p$ and $k_d$ for $K(s) = k_p + k_ds$ that meet the design specifications.
• Design a velocity control for a tape-drive servomechanism. The transfer function from current $I(s)$ to tape velocity $\Omega(s)$ (units are mm per msec per ampere) is:

$$\frac{\Omega(s)}{I(s)} = \frac{15(s^2 + 0.9s + 0.8)}{(s + 1)(s^2 + 1.1s + 1)}$$

We wish to design a type 1 feedback system so that the response to a step reference satisfies

$$t_r \leq 4 \text{ msec, } t_s \leq 15 \text{ msec, } O \leq 5\%$$

Assume a proportional-integral controller of the form $K_p(s + \alpha)/s$.

(a) Sketch out the region of the complex plane where you want to place the closed loop poles. Pick a pole location well within your designed region.

(b) Design controller zero location $\alpha$ such that the root locus passes through the chosen pole location.

(c) Sketch the root locus plot.

(d) Choose $k_p$ such that the closed loop poles are exactly at the desired location.

• Use the Nyquist stability criterion to assess the stability of the closed-loop system as a function of gain $k$ for the system with the open loop transfer function:
\[ kG(s) = \frac{k(s + 10)^2}{s^3} \]

- Plot the Nyquist plot for the transfer function:

\[ G(s) = \frac{e^{-0.5s}}{s} \]

- For the third-order plant:

\[ P(s) = \frac{50,000}{s(s + 10)(s + 50)} \]

design a lead controller such that \( \phi_m \geq 50^\circ \) and \( \omega_{BW} \geq 20 \text{ rad/sec} \) using bode sketches.

- Given a controller-plant pair \( K(s)P(s) \) that can be parameterized by:

\[ K(s)P(s) = \frac{B(s)}{s^pA(s)} \]

where \( B(s) \) and \( A(s) \) are type 0 polynomials and \( p \) is the system type, derive the table below for the \( e_{ss} \) for different reference inputs. Assume unity feedback.

<table>
<thead>
<tr>
<th>System Type</th>
<th>Step: ( R(s) = 1/s )</th>
<th>Ramp: ( R(s) = 1/s^2 )</th>
<th>Parabolic: ( R(s) = 2/s^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \frac{A(0)}{A(0)+B(0)} )</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( \frac{A(0)}{B(0)} )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>( \frac{2A(0)}{B(0)} )</td>
</tr>
</tbody>
</table>

- Consider the four open-loop (OL) bode plots of \( K(s)P(s) \) in Fig. 2. All of these OL systems are in the typical unity feedback loop and all OL systems do not have poles or zeros in the right-half plane. Fig. 3 is the closed-loop (CL) response to a unit step reference signal.
Match the OL bode plots to the corresponding CL step-response. To best prepare yourself for the exam, carefully consider:

(a) The system type and the implications of system type on the CL response.

(b) The cross-over frequency of the OL system and the corresponding CL bandwidth.

(c) The DC gain of the OL system and the implications of DC gain on the CL $e_{ss}$.

(d) The $g_m$ and $\phi_m$ measured from the OL system and implications of the stability margins on CL response.

(e) The shape of the corresponding Nyquist plot and how some (not all) information in the bode plot could be expressed in the form of the Nyquist plot.
Fig. 2. Set of open-loop bode plots.
Fig. 3. Set of closed-loop responses to a step reference.