AME 30315; Spring 2013; Midterm 1 Review (not graded)

Problems:

- 8.3.4 and 8.3.11
- 8.6.1 and 8.6.4
- 8.11
- 8.13
- 8.16
- 8.19
- 9.28
- Use partial fraction expansion to compute $x(t)$ when:

  \[ X(s) = \frac{4}{s^2 + 2s + 4} \left( \frac{1}{s} \right). \]

Use partial fraction expansion to compute $x(t)$ when:

\[ X(s) = \frac{4}{(s^2 + 2s + 4) \left( \frac{s}{20} + 1 \right) \left( \frac{1}{s} \right)}. \]

Are the responses similar? Explain whether this was expected or unexpected.

- Match the poles of a transfer function $G(s)$ to the correct impulse response $g(t)$ in Fig. 1.

  Note: you should not need to do any math to do this.

$G(s)$ pole locations:

1) $s = -2, -2$
2) $s = +\sqrt{2}i, -\sqrt{2}i$
3) $s = \frac{1}{2} + 4i, \frac{1}{2} - 4i$
4) $s = +5i, -5i$
5) \( s = -2 \)

6) \( s = -\frac{1}{2} + 4i, -\frac{1}{2} - 4i \)

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Fig. 1. Various impulse responses

- You are given the transfer function:

\[
G(s) = \frac{-100}{(s + 10)(s + 100)}e^{-0.5s}.
\]

1) Plot the bode plot for \( G(s) \)

2) What is the bandwidth of \( G(s) \)?

3) Find the analytic solution to \( |G(i\omega)| \) and \( \angle(G(i\omega)) \).
• Plot the bode plot for the transfer function:

\[ G(s) = \frac{s - 100}{(s + 10)(s + 100)}. \]

• The AD8131 op-amp you studied in Homework 3 has the transfer function:

\[ G(s) = \frac{-2}{\left( \frac{s}{2\pi \times 10^6} + 1 \right)^2}. \]

1) Find the analytic solution for \( |G(i\omega)| \) and \( \angle(G(i\omega)) \).

2) As you know, any periodic function can be written as an infinite sum of sinusoids, also called a Fourier Series. For instance, a square wave with magnitude \( A \) and period \( T \) can be written as:

\[
V_i(t) = \frac{1}{2}A + \sum_{k=1}^{\infty} \frac{A}{\pi k} (1 - \cos(\pi k)) \sin \left( \frac{2\pi k}{T} t \right).
\]

Of course it is impossible to have a sum of an infinite number of sinusoids, instead we typically approximate the periodic function by \( N \) sinusoids. Fig. 2 demonstrates a square wave approximation for \( N = 5, 50, \) and 500 sinusoids.
Given your analytic solution from Part 1, write the analytic solution for the output, $V_o(t)$, given that the input is the infinite sum in Equation (1).

3) Assuming you have the correct solution in Part 2, the actual output, $V_o(t)$, is shown in Fig. 3 for $N = 5,000$ and $T = \frac{1}{100 \times 10^6}$ sec. The ideal output, $-2V_i(t)$, is plotted as well. Describe why the sharp corners to the square wave have been rounded off.
Fig. 3. **Ideal and actual output of the AD8131.**

- Feedback problem

Figure 1. System for Problem 3.

![System Diagram]

Figure 2. Block diagram for Problem 3.

Consider the system illustrated in Figure 1.

(a) Find the transfer function from the applied force, \( f(t) \) to the position of the mass, \( x(t) \).

(b) Consider the block diagram illustrated in Figure 2. Using your answer from part 3a, determine the transfer function from the input, \( r(t) \) to the output, \( x(t) \).

(c) Let \( m = k = b = 1 \), \( R(s) = \frac{1}{s} \) and consider the two cases where \( k = 1 \) and \( k = 2 \).

   i. Which \( k \) value will have a faster rise time?

   ii. Which \( k \) value will have a greater percentage overshoot.

   iii. Which \( k \) value will have a faster settling time?

   In each case, explain your answer.
• System modeling problem

Consider the system illustrated in Figure 2.

\[ 2m \ddot{x} + ml \left( \dot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta \right) + kx = 0 \]
\[ ml^2 \ddot{\theta} + ml \ddot{x} \cos \theta + mgl \sin \theta = \tau. \]

If \( \theta \) and \( \dot{\theta} \) are small, then an approximation to the equations is

\[ 2m \ddot{x} + ml \ddot{\theta} + kx = 0 \quad (1) \]
\[ ml^2 \ddot{\theta} + ml \ddot{x} + mgl \theta = \tau. \quad (2) \]

(a) Using equations 1 and 2 determine the transfer function from the input torque, \( \tau(t) \), to the position of the mass, \( x(t) \).

(b) If the torque, \( \tau \), is applied by a d.c. motor driven by the circuit illustrated in Figure 3, determine the transfer function from the applied voltage, \( v(t) \), to the position of the mass, \( x(t) \).

• Multiple Input Problem
Problem 1

Consider a model of a two tank system containing a heated liquid, where $T_D$ is the temperature of the fluid flowing into the first tank and $T_J$ is the temperature of the liquid flowing out of the second tank. We would like the temperature of the 2nd tank to take on some desired value of $T_{2d}$.

This system of two tanks has a controllable heat input, $Q$, into the first tank. That is, a controllable amount of heat $Q$ can be added to modify the temperature of tank 1 and then, by flow exchange, the temperature of tank 2. The block diagram of the temperature control system is given below in the figure.

The time constants are $\tau_1 = 10$ seconds and $\tau_2 = 50$ seconds.

(a) Determine the temperature $T_J(s)$ in terms of $T_D(s)$ and $T_{2d}(s)$. I’m looking for one equation here: $T_J(s) = \ldots$.

(b) If $T_{2d}(s)$, the desired output temperature from tank 2, is instantaneously doubled from $T_{2d}(s) = 1/s$ to $T_{2d}(s) = 2/s$, where $T_{2d}(s) = 1/s$, determine the transient time domain response of $T_J(t)$ when $G_C = K = 500$.

(c) Find the steady state error $e_{ss}$ for the system of part b, where $E(s) = T_{2d}(s) - T_J(s)$. 