Problems:

- 9.5
- 9.13
- 9.17.1–3
- 9.19
- 9.24

1. A stable plant is inherently more stable than an unstable plant when in a feedback loop. Prove that a stable first-order or second-order plant with positive gain, \( K > 0 \), is stable for all PD-Control designs (\( k_p \) and \( k_d > 0 \)):

\[
P_1(s) = \frac{K}{s + a}; \quad P_2(s) = \frac{K}{(s + a_1)(s + a_2)}; \quad \text{or} \quad P_3(s) = K \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

Also, prove that this is not true for unstable plants.

2. Consider the system shown in Fig. 1, which consists of a prefilter and a unity feedback system.

1) Determine the transfer function from \( R \) to \( Y \).

2) Determine the steady-state error due to a step input.

3) Discuss the effect of different values of \( (K_r, a) \) on the system’s response.

\[
\begin{align*}
K_p \frac{s+1}{s+a} & \quad + \quad A \\
\frac{A}{s(\tau s+1)} & \quad \rightarrow \quad Y
\end{align*}
\]

Fig. 1. Feedback system with a pre-filter.
• **Sensor Problem** In Fig. 2 you wish to control the position of a motor with a plant transfer function:

\[ P(s) = \frac{1}{s(s + 4)}. \]

Your sensor has the transfer function:

\[ P(s) = \frac{K}{s + 2}. \]

Plot the root locus for the poles of the closed-loop transfer function as \( K \) goes from 0 to \( \infty \). At what value of \( K \) does the system become unstable.

Fig. 2. Feedback system with a sensor with dynamics

• Consider the open-loop system:

\[ G(s) = \frac{1}{s^2 + 4s + 3} \]

where the poles are the values of \( s \) such that:

\[ 1 + kG(s) = 0. \]
1) Sketch the root locus for this transfer function.

2) Indicate the section of the root locus for which the system under unity feedback will have an overshoot of less than 5% if the input is a step.

3) Indicate the sections of the root locus for which the system will have a rise time of approximately 1 second if the input is a step.

4) Will it be possible to chose a $k$ such that the closed loop system will have an overshoot less than 5% and a rise time less than 1 second?

- Systems that are open-loop unstable require feedback to stabilize them. Systems that are non-minimum phase are stable, but have other undesirable properties. Some feedback control researchers have argued that it is better, in terms of stability, for a system to be open-loop unstable than for it to be non-minimum phase (Hoagg and Bernstein, *Control Systems Magazine*, 2007). Given what you proved in Homework 7, problem 4, and what you have learned about root locus analysis, why do you think some researchers have made this claim?

- Root Locus Problem
The open loop transfer function of a system to be controlled is shown below

\[
\begin{array}{c}
\text{u} \\
\downarrow \\
\frac{5}{s(s+4)} \\
\downarrow \\
\text{y}
\end{array}
\]

Here, \( u \) is the system input and \( y \) is the system output. A unity feedback control law has been proposed of the form:

\[
u(t) = -k_4(y - r) - k_3\dot{y} - k_3 \int_0^t (y - r) dt
\]

where \( y \) is the system output and \( r \) is a reference signal. For this exam problem, \( k_2 = 0 \) and \( k_1 = k_3 = K \).

(a) Draw the root locus of the system as \( K \) goes from 0 to infinity including the following, if applicable:

1. Open Loop Poles and Zeros
2. Asymptotes
3. Breakaway/Break-in points
4. Departure and Arrival angles
5. jo axis cross over points

You must show all calculations to receive full credit. You can use the axes provided on the following page.

- Pole Placement / Root Locus Problem
The root locus plot for

\[ G(s) = \frac{1}{s^2 + 2s + 2} \]

is illustrated in Figure 2. For this \( G(s) \) and the feedback system illustrated in Figure 1, determine

(a) an approximate value of \( K \) that will yield a maximum percentage overshoot of \( M_p = 23\% \);
(b) an approximate value of \( K \) that will yield a rise time of 0.9 seconds; and,
(c) an approximate range of values of \( K \), if possible, that will satisfy both criteria, i.e., a maximum percentage overshoot less than 23\% and rise time of less than 0.9 seconds.

Where possible, it is appropriate to determine approximate values of pole locations by simply referring to Figure 2.

(20 points)

![Root Locus Plot](image)

*Figure 2. Root locus plot for Problem 2.*

- PID Control Design Problem
Consider the system illustrated in Figure 1.

(a) Find the transfer function from the applied force, \( f(t) \) to the position of the mass, \( x(t) \).

(b) Consider the block diagram illustrated in Figure 2. Using your answer from part 3a, determine the transfer function from the input, \( r(t) \) to the output, \( x(t) \).

(c) Let \( m = k = b = 1 \), \( R(s) = \frac{1}{s} \) and consider the two cases where \( k = 1 \) and \( k = 2 \).

i. Which \( k \) value will have a faster rise time?
ii. Which \( k \) value will have a greater percentage overshoot.
iii. Which \( k \) value will have a faster settling time?

In each case, explain your answer.

- **For those who want a challenge**

Consider the plant:

\[
P(s) = \frac{600}{(10s + 1)(60s + 1)}
\]

in unity feedback with \( P \)-control with a gain of 1 and a variable delay, \( K(s) = e^{-\tau s} \).

Typically we estimate a time delay by the first two terms of the Taylor Series expansion:

\[e^{-\tau s} \approx 1 - \tau s.\]
Plot the root locus as $\tau$ ranges from 0 to $\infty$. At what delay time $\tau$ does the system become unstable. *Hint:* you will have to use the negative (0 deg) root locus instead (ftp://www.ece.lsu.edu/pub/ggu/Lecture_notes/EE4580/note3.pdf).