Investment, Externalities, and the Boundaries of the Firm

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Abstract
Bilateral negotiation has been studied extensively in economics, particularly when it is done in the presence of externalities. However, an analysis of investment decisions made during these situations remains absent. This paper addresses this deficiency by studying a principal who trades with many agents; the principal may make investments that augment market activities. Externalities among the agents may distort trade and investment. We find that if investment undertaken in a market is distorted, then trade in the market is also distorted, but the converse of this statement does not hold in general. Also, the types of externalities which distort outcomes differ depending on whether contract negotiations are held in public or private. This suggests that outcomes under private negotiations may be efficient while outcomes under public negotiations may not. Importantly, we demonstrate that agents can counteract distortions by merging into a firm. Thus, firms may lead to efficiency gains even when there is no hold-up problem, a finding which runs counter to standard theories on the boundaries of the firm.
1 Introduction

Research in economics has shown that there is utility in analyzing situations where market activities affect interactions among potential sellers or buyers. There are many examples of these types of situations in the world. Sales transactions between upstream and downstream businesses, the sale or licensing of important technological innovations to businesses, the sale of nuclear weapons between countries, the provision of a public good by a private party, issues related to mergers and acquisitions, and the hiring of a “superstar” employee are all examples of games where sales transactions may create externalities among sellers or buyers. Moreover, these games have implications regarding the organization of industry, optimal mechanism designs, and efficiency.

The implications of externalities on investment have received less attention. Of course, the effects of contracting environments on investment decisions have been studied in great depth by economists, particularly in relation to the “hold-up” problem (Holmstrom and Roberts, 1998). However, such studies often argue that issues related to asset allocation (De Meza and Lockwood, 1998), outside opportunities (Hart, 1995) or incomplete information (Tirole, 1988) explain why investment decisions differ from first-best decisions. Explanations appealing to externalities are seldom seen.

This paper asks two research questions. First, how do externalities affect investment decisions made during bilateral negotiations? We find that externalities can distort trade and investment decisions in a market. Second, can firms play a role in counteracting the effects of these distortions? We find that (at least sometimes) they can.

To answer these research questions, we generalize a model of bilateral negotiation presented by Segal (1999) to include investment decisions. The model depicts a principal who undertakes exchanges with many agents; externalities exist between agents in a manner described below. Unlike Segal’s model, this model allows the principal the ability to make investments which can

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1 Bilateral negotiations will be described further in what follows, but in general a contract between a seller and a buyer is said to be bilateral if the price named in the contract does not explicitly depend on actions taken by other parties in the market.
augment the quality of the product for sale. This model is in fact quite general and can be stylized to describe any of the situations mentioned at the start of this paper. The results of this paper are also applicable to both games where negotiations with individual agents are held in public and games where negotiations are private.

In answering these research questions, this paper serves as a bridge between the two areas of research mentioned above; it introduces investment decisions to models of contracting with externalities and it introduces externalities to models of investment. Why is this important? Aside from showing that issues in these two literatures can be analyzed using one model, this paper makes at least three other contributions.

First, this paper's model describes real-world situations that involve externalities and investment. Any of the above real-world examples could easily be enriched to provide the seller of an item opportunities to improve or affect the quality of the item for sale, but it may be best to consider a specific example. Consider a scenario similar to one discussed in De Fraja (1999). An entrepreneur designs computer programming products for two different buyers: a hardware manufacturer and a software manufacturer. These products create positive externalities between the two buyers: even if only one buyer trades with the entrepreneur, the market demands both more software and more hardware. The entrepreneur may choose the amount of resources to invest in her computer programming products. It seems that in this situation the existence of an externality between the two buyers may lead the entrepreneur to choose a level of investment different from the level that maximizes total market surplus. But exactly how would this distortion manifest itself? Current research does not say.

Second, this paper demonstrates that the important comparative-static results derived in Segal's work can be extended (with some modifications) to investment decisions to make robust predictions about investment. This is important because it is not generally easy to infer from the presence of externalities how investment would be distorted, which is probably why current research cannot tell us what the entrepreneur will do in the above example. One may guess that
if there is a positive externality in the market and investment compliments trade then too little investment will be made, but this is not necessarily true. In fact, it is possible that because of the externality the intreprenuer overinvests in her computer programming products. An example depicting this type of situation is given below. Understandably, it seems that this sort of ambiguity has prevented a serious study of the efficiency of investment decisions and their relations to externalities among negotiating parties.

We find that the type of externalities which distort outcomes are different under various information settings. Under public offers, inefficiency may occur from the effects externalities have on reservation payoffs. However, with private offers only the externalities present at efficient levels of trade and investment distort market outcomes. This suggests that under some types of externalities a game with privately observed offers may lead to the best-outcome while a game of publicly-held negotiation may not; an example of such a scenario is provided. It can be the case that trade may be greater than efficient while investment may be less than efficient; whether or not investment and trade “move together” depends on whether or not investment is complementary to trade.

Finally, and perhaps most importantly, this paper shows that the distortions caused by externalities may be abrogated by the presence of a firm in the market. It is well known that externalities may be considered a manifestation of market incompleteness. In fact, models with externalities could also be considered models with a special type of incomplete contract. This paper shows that firms may lead to efficiency gains in these type of situations.

The cause of a contracts' incompleteness should not be underestimated. In our model, firms can restore efficiency even if investment is not in the model at all. In contrast, traditional models of the hold-up problem (such as Grossman and Hart, 1986, and Hart and Moore, 1990) rely on a concept of investment of some sort. To quote Holmstrom and Roberts' well-known paper, theories on the boundaries of the firm “focus on the role of ownership in supporting relationship-specific investments in a world of incomplete contracts and potential hold-ups.” Our paper shows that
a very common type of contractual incompleteness, externalities, leaves room for the firm even without investment.

Consider the example of insurance companies. Suppose there was a spot market for insurance where some agents sold life insurance while other agents sold health insurance and so on. A customer seeking to purchase insurance from different agents could do so in a way which fails to maximize surplus in the market. This is because there are externalities among the different insurance agents. For instance, a life insurance agent’s expected profit from offering the customer coverage could depend on whether or not the customer has other types of insurance.

This paper’s findings suggest that in markets with externalities between insurance agents it may be efficient for one firm to offer different types of insurance instead of having an insurance spot market. A single firm offering different types of insurance can internalize the externalities that exist between different insurance agents. 2

Furthermore, the “merger” described here has nothing to do with asset acquisition; it could be efficient for various third parties to simply have one central negotiator offer all of their various products at once. There is evidence of similar behavior with insurance companies. Often, autonomous third parties will join together to offer one menu of insurance options to consumers through a central negotiating party such as a financial services institution (Glassman, 2000). It may be possible to predict similar behavior in the insurance market using traditional theories of the firm. However, in some cases the traditional theory can make predictions different from a theory based on externalities and such differences could be tested to improve our understanding of the role firms play in markets.

The rest of this paper is organized as follows: the next section provides the model of bilateral negotiation to be used. Section 3 then studies investment decisions and externalities under a public offers setting; sufficient conditions are derived for comparing first-best outcomes to the outcomes

2 In fact, it is common to observe firms in the insurance market offering different types of insurance (Glassman, 2000).
generated by the market. Section 4 expands these results to situations where negotiations between
the principal and an agent are held in private. Section 5 examines the role that .rms can play in
this model. The last section concludes.

2 The Model

Consider a setting where a single principal has the opportunity to transact with n other agents.
The principal could be a seller oering products to many buyers or a buyer purchasing a product
from many sellers; it does not matter. We denote the set of agents as N. The trade which occurs
between the principal and agent i will be denoted \( x_i \in \mathbb{R}_+ \) where \( \mathbb{R}_+ \) is a compact subset of the
nonnegative real numbers and \( 0 \leq x_\mathbb{R}_+ \). The total trade undertaken by the principal will be the
vector \( x \). The principal can also choose to make investments \( y \in \mathbb{R}_+ \) where \( \mathbb{R}_+ \) is again a compact
subset of the nonnegative real numbers; this investment aects the quality of the trade between
the principal and the agents. We will denote the cost of investing \( g(y) \). For the rest of the
paper we consider \( y \) to be a scalar, but what follows could be extended to cases where it is not.
However, our analysis does assume that only one party, the principal, invests.

When \( x_i > 0 \) surplus is created: Specifically, agent i’s total gain from trade is \( u_i(x; y) \); and
the principal’s total gain from trade with all agents is \( \sum_{i=1}^{P} t_i f(x) \) \( g(y) \); where \( t_i < 0 \) is a monetary
transfer. Externalities are present in this market since the surplus created from a transaction
with agent “i” depends not only on that transaction but also on all the other transactions in the
market. This may affect the outcomes the market produces. Situations where there is one seller
and n buyers would be represented by the seller selling \( x_i \) to buyer i with \( f(.) \) increasing in x as
the cost function and \( t_i > 0 \) as the price. Alternately, the model could depict a situation where
there is one buyer and n sellers and the buyer buys \( x_i \) from seller i, and so \(-f(.) \) would represent
the buyer’s surplus and \( t_i < 0 \) would be the price paid to each agent. While this model does not
allow for investment to aect \( f(.) \), this is a simplification and the model could be extended to this
generality.
We will use the concepts of increasing differences and decreasing differences when describing the model's implications and we define them next:

**Definition:** There exist increasing differences (decreasing differences) between investment and surplus if \( u_i(x^A; y) - u_i(x^B; y) \) is nondecreasing (nonincreasing) in \( y \) for all \( i, y \) and \( x \) with \( x^A \geq x^B \).

Increasing differences means that the marginal surplus from trade with a given agent is non-decreasing with respect to investment. This might be appropriate when investment actions are complementary to the trade between agents. Decreasing differences may occur when investment actions substitute for trade between agents. Alternately, we may see decreasing differences where our "investment" could be some sort of unprofitable action. For example, if the principal here were selling a public bad that created pollution, investment could be costly actions taken to make the product less polluting. If such actions lowered the marginal surplus from trade, then investment would have decreasing differences.

We denote the efficient or first-best solutions in this model as:

\[
M^* = \arg\max_{x,y} \{ \sum_i u_i(x; y) - f(x) - g(y) \}
\]

where \( x \) and \( y \) are elements of their respective domains as described above. Equilibrium outcomes are distorted if they are not first-best. We assume that a solution to the above program exists (although it need not be unique).

This model is similar to the one used by Segal (1999). In fact, if we assume that \( u_i(x; y^A) = u_i(x; y^B) \) \( 8 \), \( y^A \), and \( y^B \) and that \( 0 \leq \bar{A} \) and that \( g(0) = 0 \), then the following analysis would be just like Segal's where we would note that in both the first-best outcome and the market outcomes the level of investment made is \( y = 0 \). Moreover, Segal's model itself may be stylized to analyze a number of specific situations studied previously, for example vertical contracting.

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3 The binary relation \( \bar{A} \) in \( x^A \), \( x^B \) is a component-wise relation so that \( a_i \geq b_i \) for every element \( i \) in vectors \( a \) and \( b \).

3 Externalities and Investment: Public Offers

This section considers the following game of public offers. The principal simultaneously makes an investment decision and offers each agent a take-it-or-leave-it offer \( f(x_i; t_i) \) for \( i = 1, n \). Both the investment decision and the take-it-or-leave-it offers are (for now) common knowledge. Next, the agents simultaneously decide whether or not to accept their offers. If an agent accepts an offer he "pays" the principal the transfer \( t_i \) and enjoys the surplus created by the project. If an agent does not except an offer his payoff is \( u_i(0; x_i) \) and he pays the principal a transfer \( t_i = 0 \). Notice that in order to isolate the effects of externalities on investment we will assume that the investments the principal makes do not directly affect the payoffs of agents with whom no trade is conducted.

However, contracting is bilateral in the sense that the price an agent faces is independent of the actions other agents take.

\[4\] Segal (1999) includes a discussion of the application of this model to the situations covered by these papers and lists many more related studies.

\[5\] De Meza and Lockwood's model focuses primarily on asset ownership and most of their analysis is in fact outside the range of this paper.

\[6\] Also, we assume that if the principal does not trade with any of the agents her payoff is zero, and so the investment has no benefit to the principal unless trade occurs with one of the buyers in the market (of course the principal would still have to realize the cost of investments made in such a situation).
We will consider the principal's preferred Subgame-Perfect Nash Equilibria (SPNE). Since the principal is free to offer \((x_i; t_i) = (0; 0)\) to any agent wlog we consider equilibria where all agents accept their offers.

Consider the principal's problem, which is

\[
\max_{x \in \mathbb{A}; y \in \mathbb{A}} \sum_{i} t_i \cdot f(x) \cdot g(y)
\]

where we let \(\mathbb{A} = \mathbb{A}_1 \cdot \mathbb{A}_2 \cdot \cdots \cdots \cdot \mathbb{A}_n\). A necessary condition that a Nash Equilibrium must satisfy is

\[
\text{Proposition 1: If } u_i(0; x_i) \text{ does not depend on } x_i \text{ for all } i, \text{ then } M = M^*.
\]

This individual rationality constraint states that no agent in equilibrium would trade with the principal if refusing to do so strictly increased the agent's payoff. In the principal's preferred SPNE this constraint must bind; otherwise the principal could increase the size of the transfer holding all else equal and increase profits. Solving out for transfers using this binding constraint, we may rewrite the principal's problem as

\[
\max_{x \in \mathbb{A}; y \in \mathbb{A}} \sum_{i} u_i(x; y) \cdot t_i \cdot u_i(0; x_i) \cdot f(x) \cdot g(y)
\]

The set of solutions to this program is denoted \(M\). The essential difference between the principal's problem and the first-best problem is the inclusion of reservation payoffs in the objective function. Intuitively, the principal includes reservation utilities in the objective function because she recognizes that her investment and trade decisions affect an agent's reservation payoff, which in turn affects his negotiation position. Of course, if the agent's reservation utility is unaffected by the trade done with other agents, the principal does not have any incentive to deviate from the first-best solution. 

\footnote{This is the same concept used in Segal (1999). Segal (1998) notes that there may be multiple equilibria of this type of game.}

\footnote{This proposition is analogous to proposition 1 in Segal (1999); the proof is trivial.}
Of course, it could be the case that an agent’s reservation utility is affected by the trades other agents make and these externalities could lead to distortions. Many of the situation described at the beginning of this paper make assumptions about the monotonicity of these externalities; they are either positive or negative. The following definition makes this notion concrete.

Definition: There exist positive (negative) externalities on nontraders if \( u_i(0; x_i) \) is nondecreasing (nonincreasing) in \( x_i \) for all \( i \) and all \( x \).

One may suspect that our assumptions on complementarity (e.g., increasing differences in investment) and the monotonicity of externalities suffice to describe how the principal’s decisions will differ from the first-best. This is in fact not true, as the following example shows:

EXAMPLE 1: For this example we use a model with increasing differences in investment and positive externalities. One might think both effects working together would lead to the principal both under-selling and under-investing, because as trade decreases agents have lower reservation payoffs and because investment is complementary to trade. But neither outcome holds in general.

There are two agents \((n = 2)\) and the surplus for agent \( i \) has the form \( u_i(x; y) = a_i x_i y + b_i x_i - c_i x_i^2 + d x_i x_i \) where \( a_i; b_i; c_i > 0 \). We let \( f = 0 \) and the cost of investment is \( g(y) = y^2 / 2 \). We will assume that our choice variables are continuous over the non-negative real numbers.\(^9\)

Then there exist increasing differences since \( a_i > 0 \) and externalities on nontraders are positive since \( b_i > 0 \). Notice that the parameter \( d \) could be viewed as an additional type of externality; this will turn out to be important in what follows.

We can write our objective functions as

\[
\max_{x_1; x_2; y \geq 0} x \left( a_1 x_1 y + b_1 x_1 - c_1 x_1^2 + d x_1 x_1 \right) + y^2 / 2 + z b \ x_1
\]

Looking at the last term, note \( z = 0 \) corresponds to the first-best problem and \( z = 1 \) corresponds to the principal’s problem. We assume \( c_1 c_2 > 4 d^2 \) and \( \xi^2 a_2^2 c_2 + a_2^2 c_1 + 4 d^2 + c_1 c_2 + 4 d a_1 a_2 < 0 \); these along with our assumptions on positive coefficients ensure a concave objective function. Our

\(^9\) Since we assume compact domains for our choice variables, for this example we may suppose that our variables are bounded above and below by a large number and a small number, respectively.
The rst-order conditions yield: a) \( a_1 y + b_1 (1_i z_i) x_i + 2d x_i = 0 \) for \( x_i \) and b) \( y = a_1 x_1 + a_2 x_2 \) for \( y \). If aggregate trade is at its optimal level, investment will be at its optimal level also. Letting \( x_i^n \) denote the rst-best solution and \( x_i^S \) the principal’s solution, we can manipulate the rst order conditions to show

\[
x_i^S = x_i^n + \frac{1}{\xi} b_i (2d + a_1 a_1) + b_1 (c_i x_i^n + a_2^2) : \]

Using our rst-order condition for \( y \) we see

\[
y^S = y^n + \frac{1}{\xi} (a_1 b_1 + a_2 b_2) (2d + a_1 a_2) + a_1 b_2 (c_2 x_i^n + a_2^2) + a_2 b_1 (c_1 x_i^n + a_2^2) : \]

where the term on the right represents the difference between the rst-best level of investment and the principal’s profit-maximizing level. If we let \( a_1 = 3; a_2 = 2.55; c_1 = c_2 = 10; \) and \( d = -4.75 \) we are left with

\[
y^S = a_1 x_1^S + a_2 x_2^S = 3x_1^n + 2.55x_2^n + 13.5 (5.775 b_1 x_1 - 3 b_2) : \]

Again, the right-most term represents the distortion from externalities: The key here is that this term can be either negative or positive when \( b_1; b_2 > 0 \): The rst case would follow the standard intuition of how investment (and trade) behave when there are positive externalities on nontraders present. In the second case investment and trade are above their optimal levels; the principal trades and invests “too much.” In both cases, however, externalities on nontraders are positive and there are increasing returns from investment.\( y \)

What is going on in this example? The culprit is obviously \( d \); which creates a second type of externality.\(^{10}\) This new externality can offer incentives to the principal that differ from the externalities captured in the \( b \) parameters. Interestingly, we show below that the following two fairly general conditions ensure that the incentives created by externalities will not be in conflict with each other.

\(^{10}\) It is easy to verify that if \( d = 0 \) then \( y^S = y^n \), which is the intuitive outcome under positive externalities. However, less drastic restrictions on \( d \) would also restore our intuition (namely, any restrictions satisfying condition A, which we define below).
CONDITION D: Either $\hat{A}_i = [0; x_i]$ or $\hat{A}_i = \{ k \in \mathbb{N} : k = 0; 1; \ldots; k \in \{ 0; 1; \ldots; n \} \}$.

Condition D (the Domain condition) says that the increments of trade are the same for all agents, but trade could be in finite increments or continuously measured.

CONDITION A: $P \cdot u_i(x; y) - f(x) - g(y) = A(P \cdot x_i; y)$.

Condition A (the Aggregation condition) says that aggregate surplus is a function of aggregate trade. This condition may be satisfied, for example, by functions of the form $u_i(x; y) = x_i \cdot (X^\circ) \cdot (Y) + \cdot (X)$, where $X = P \cdot x_i$.

With these conditions in hand, we can begin to make robust comparisons between the principal’s investment and trade decisions and the surplus maximizing outcomes. By the “aggregation method” described in Milgrom and Shannon (1994) we can rewrite the principal’s objective function under condition A as

$$
\max_{x: \mathbb{R}, y: \mathbb{A}} A(X; y) - R(X)
$$

(4)

where $R(X) = \min_{x: \mathbb{A}} \int \cdot u_i(0; x_i) : x = X \cdot g$ is the minimum sum of reservation utilities consistent with the aggregate amount of trade $X$.

This version of the principal’s objective function and the following lemma will together provide powerful insights into the effects of externalities.

Lemma 1: If condition D holds and externalities are positive (negative), then $R(X)$ is nondecreasing (nonincreasing) on its domain.

Proof: See appendix.

Condition D is a key element of this Lemma. To see why, we use an example from Segal (1999). Suppose that there are two agents ($n = 2$) and that agents 1 and 2 are only allowed to trade in multiples of 3 and 5, respectively. Then the principal can achieve $X = 10$ from trading with agent 2 only, but to implement $X = 8$, trade with both agents would have to occur. With

$\hat{A} = \{ x_i : x_i \in \mathbb{A} \}$.
positive externalities, this increase in trade with agent 1 would increase agent 2’s reservation payoffs and potentially the sum of reservation payoffs.

It is possible that the principal’s optimal solution to her profit-maximization problem is not single valued, and that the surplus maximizing levels of trade and investment are not single valued either. Rather than impose assumptions on the model which would ensure single-valued solutions, we will compare the set of profit-maximizing solutions ($M$) and the surplus-maximizing solutions ($M^*$) using the strong induced set order which is described in Milgrom and Shannon (1994). For two sets $A$, $B$, we say $A \preceq B$ if whenever $a \in A$; $b \in B$; and $a \leq b$ we also have $b \in A$; and $a \in B$.

Throughout the paper we will apply the strong induced set order on the following sets:

Definition: $M_x = \{ x | P_x(i) = X; (x;y) \in M_g \}$ and $M_y = \{ y | (x;y) \in M_g \}$; and similarly for $M^x$ and $M^y$.

Armed with these concepts, we are ready for the first key result of this paper:

**Proposition 2** If Conditions D and A hold and there are increasing differences in investment, then with negative (positive) externalities on agents $M^x \preceq (\preceq) M^y$ and $M^y \preceq (\preceq) M^x$.

Proof: See appendix.

There are two types of intuition for this proposition, mathematical and economic. The mathematical intuition is that by construction the choice variable domain is a lattice, in which case the supermodular properties of our objective function lead to robust comparative statics. The function’s supermodularity is proven in the appendix.

The economic intuition behind this proposition is more interesting. Conditions A and D ensure that all present externalities work in harmony with one another. If aggregate surplus is a function of aggregate trade, then the principal will not be distracted by conflicting incentives among externalities because there are no conflicting incentives. When externalities work in harmony the outcomes described by the proposition are intuitive. For example, when externalities are negative and there are increasing differences in investment, the principal has an incentive to
undertake more transactions than would be socially optimal in order to improve her negotiating position. Since investment is complementary to trade, investment rises above its optimal level as well.

Furthermore, this proposition shows that studying the relationship between externalities and investments in bargaining situations is both possible and instructive to the study of industrial organization. There is a large, even enormous literature in economics focusing on problems with investment in negotiating situations. This proposition and the ones that follow show that ine¢ient investment may occur without appealing to the importance of asset allocation, the full-information setting, the existence of outside opportunities or even the notion that investment itself may directly impose externalities. As we will see later, this paper shows how the existence of rms can lead to e¢ciency gains even when the standard explanations do not apply.

We may be interested in situations where investment and trade diverge. For example, if a producer of a good which creates pollution can take costly investment actions to make her product cleaner, we might expect to see her over-produce and under-invest at the same time. The following proposition considers such situations.

Proposition 3 If Conditions D and A hold and there are decreasing differences in investment, then with negative (positive) externalities $M_x \leq (1 \text{ ) } M_x^*$ and $M_y \leq (0 \text{ ) } M_y^*$.

Proof: See appendix.

Proposition 3 describes situations where some choice variables are potentially larger than is e¢cient, while others are smaller. In this sense it is different from Segal’s (1999) study where outcomes of trade pro-.ies alone are analyzed. Coupled with Proposition 2, these two propositions make predictions regarding market outcomes robust to the presence of externalities. Importantly, these propositions also highlight that the mere existence of (say) positive externalities does not lead to any hard-and-fast rule for investment; in fact, the situations depicted in each proposition have investment going in opposite directions. Our intuition is that not only the sign of the externalities, but also whether or not investment is complementary to trade, indicate how trade
and investment respond to externalities.

Propositions 2 and 3 focus on $M_x$ and $M_y$ separately, but one may be interested in how trade and investment move together. The following proposition shows that we can make inferences about how trade and investment move together in some situations.

**Proposition 4**

Under Conditions D and A; suppose that there is an equilibrium $(x, y)$ where aggregate trade $X = \sum x_i$ is efficient. Then equilibrium investment $y$ is also efficient:

**Proof:** See appendix.

Notice that this proposition also implies that if investment is distorted in equilibrium then aggregate trade is also distorted. The intuition is that distortions come from the principal's efforts to affect agents' reservation utilities (and hence their bargaining positions). These reservation utilities are only a function of aggregate trade; they are not directly affected by investment. Thus, if the principal does not gain from distorting trade she will not gain from distorting investment either. But the converse is not true: it could be that for many levels of trade, some efficient and some not, that the same level of investment maximizes the principal's objective function. This means that if trade is distorted we cannot infer whether or not investment is distorted too.

4 Externalities and Investment: Private Offers

There are a number of interesting contracting situations where negotiations between the principal and an agent may be private. For example, the literature on vertical contracting often considers a game where the principal makes offers simultaneously to agents, but agents only observe their own offer and not the offers made to the other agents. This subsection considers a contracting game much like the game above but now agents privately observe their own trade offer.\(^{12}\) There are two stages: in stage one, the principal publicly announces an investment decision and makes an offer to each agent $i$, $(x_i; t_i)$; which the agent privately observes. In stage two, agents simultaneously accept or reject. We consider PBE equilibria of this game. Obviously, actions taken in step

\(^{12}\) This game is therefore a game of complete but imperfect information.
two may be influenced by agents' beliefs about the contracts offered to other agents. To avoid a muddled situation with many equilibria, we follow Segal (1999) and McAfee and Schwartz (1994) and assume agents hold “passive beliefs,” where even after observing unexpected behavior from the principal, an agent still believes that other agents received their equilibrium offers. Furthermore, the inclusion of beliefs necessitates a modified definition of externalities. We present this definition next.

**Definition**: Externalities on efficient traders are positive (negative) if for all \((x^n; y^n) \in \mathcal{M}^n\); and each agent \(i\), \(u_i(x^n_i; \bar{x}_i; x_j^n; y^n)\) is nondecreasing (nonincreasing) in \(x_j^n \in \bar{A}_i\).

Since the principal can always offer \((0,0)\), again we focus on situations where all agents accept their offers.

We will assume that an equilibrium outcome \((\hat{x}, \hat{t}, \hat{y})\) exists. Let us consider the gain the principal may realize by deviating from an equilibrium outcome. The principal’s problem is

\[
\max_{x \in \bar{A}, t \leq n, y \in \bar{A}} \sum_{i} u_i(x_i; \hat{x}_i; y) - f(x) - g(y)
\]

subject to \(u_i(x_i; \hat{x}_i; y) \geq t_i\). Note that \((\hat{x}, \hat{t}, \hat{y})\) is an equilibrium if and only if it solves this program. As before, all participation constraints will bind if the principal is maximizing profit. By assumption agents take \(u_i(0; \hat{x}_i)\) as given, and we therefore know that

\[
\hat{x}; \hat{y} \geq \arg \max_{x \in \bar{A}, y \in \bar{A}} \sum_{i} u_i(x_i; \hat{x}_i; y) - f(x) - g(y)
\]

Let \(\mathcal{Y}\) denote the set of such trade and investment decisions. The problem the principal faces here is very similar to the problem that she faced with public offers except now she has an additional constraint on her program: in equilibrium her actions must be best-responses given the beliefs of the agents. This suggests that the principal cannot do better with private offers than with public offers where she has the power of commitment.

Obviously, there may be elements in \(\mathcal{Y}\) which differ from the first-best outcomes which solve
Interestingly, however, if externalities are absent at any efficient trade and investment profile, the principal’s contracts will always lead to an efficient outcome, even if externalities exist at other trade and investment profiles.

**Proposition 5** If there exists \((x^*, y^*) \in M^n\) such that \(u_i(x^n_i; x_{\bar{i}}; y^n)\) does not depend on \(x_{\bar{i}}\) for all \(i\), then \(\forall \mu \in M^n\):

Proof: See appendix.

This suggests that with private offers and externalities the investment decisions in equilibrium may still be surplus-maximizing. Furthermore, it is entirely possible that there are externalities for nontraders (i.e., trade affects reservation payoffs) but externalities may be absent at an efficient trade profile. In that case, Propositions 2, 3, and 5 suggest that the outcome under public offers may be inefficient while the outcome generated under private offers is always efficient. Both the nature of the relevant externalities and the assumption of passive beliefs play important roles in this intriguing outcome. Under private offers the principal seeks rent from the agents by strategically moving away from the first-best solution—the nature of this distortion is described below. But if externalities on efficient traders are absent, agents credibly anticipate an efficient outcome; the principal’s ability to distort is annulled. In contrast, under the public offers setting, agent’s cannot remain ignorant of the externalities they face should they choose not to trade and this provides an opportunity for distortion. These concepts are illustrated in the following example.

**Example 2:** Suppose that there are again two agents \((n = 2)\) and they are buyers. Here \(x_i \in [0; 1]\) and the surplus consumed by buyer \(i\) is described by

\[ u(x_i; x_{\bar{i}}; y) = yx_i + (1 - x_i)x_{\bar{i}}. \]

Thus, investment is complementary to trade and there are positive externalities from trade. However, as the amount of trading with agent \(i\) increases the size of the externality consumed by agent...
i falls ceterus paribus. When the maximum amount of trade with agent \(i\) is undertaken \((x_i = 1)\), agent \(i\) is not affected by trade with the other agent at all.

The principal faces a cost of investment \(g = \frac{1}{2}y^2 + 2y\) and \(f = \frac{3}{2}(x_1^2 + x_2^2)\). Let us consider the rst-best outcome, the equilibrium outcome under public offers and the equilibrium outcome when aggregate trade offers are observed privately.

First we consider the rst-best outcome, the outcome which solves (1):

\[
\max_{x_1, x_2, y} y(x_1 + x_2) + x_1 + x_2 \quad 2x_1x_2 \quad \frac{3}{2}(x_1^2 + x_2^2) \quad \frac{1}{2}y^2 + 2y
\]

It is straightforward to show that the solution to this problem is \(x_1 = x_2 = 1\) and \(y = 4\). Social surplus is maximized when trade with both agents is at its maximum. Importantly, this is a level of trade where externalities are absent. In other words, we have externalities on non-traders, agents for whom \(x_i = 0\); but we do not have externalities on efficient traders, agents for whom \(x_i = 1\).

Next we will consider the principal’s problem under public offers. Using (3), the principal’s problem is

\[
\max_{x_1, x_2, y} y(x_1 + x_2) + x_1 + x_2 \quad 2x_1x_2 \quad \frac{3}{2}(x_1^2 + x_2^2) \quad \frac{1}{2}y^2 + 2y
\]

The principal includes reservation utilities in the objective function, making this problem different from the rst-best problem. Externalities are positive and the principal has an incentive to decrease the amount of trade undertaken with each agent below the rst-best level; doing so lowers the agent’s reservation utility and therefore weakens his bargaining position. In this case it is again easy to show that the solution to the problem is \(x_1 = x_2 = \frac{2}{3}\) while \(y = 3\frac{1}{3}\); both trade and investment are below the surplus-maximizing levels.

Finally, we consider the principal’s problem with private offers—that is, agent \(i\) observes \(y\) and \(x_i\) but does not observe the offer made to the other agent. Using (6) the principal’s problem is

\[13\] The reader may be wary since the cost of investment may be negative. This functional form is chosen for convenience and similar outcomes can be derived using a more traditional cost function. Also, surplus in this example does not satisfy Condition A but Proposition 4 does not depend on Conditions A and D.
now
\[
\max_{x_1, x_2, y} y(x_1 + x_2) + (1 - x_1)x_1 + (1 - x_2)x_2 \cdot \frac{3}{2}(x_1^2 + x_2^2) + \frac{1}{2}y^2 + 2y
\]

where $x_i$ is agent i's belief concerning the offer made to the other agent. This problem is clearly different from the first-best problem. However it turns out that the equilibrium outcome is $x_1 = x_2 = 1$; and $y = 4$. This is the outcome that maximizes surplus. This is no surprise in light of Proposition 5, which states that if externalities on efficient traders are absent any equilibrium with private offers then equilibrium trade and investment must maximize surplus. Again, the intuition is that agent's hold passive beliefs and therefore do not give the principal an incentive to introduce distortions by offering outcomes different than the first-best outcome. However, agents do not have the opportunity to remain credibly ignorant under public offers and the principal therefore has an incentive to seek rent. Private information in this setting is good—it protects the agents from knowledge that can hurt their negotiating position.

If externalities are present at all efficient trade profiles, then distortions may occur regardless of whether or not trades are publicly observable. As with the full-information setting, we cannot in general predict efficient outcomes for trade and investment. Once more we must appeal to Conditions A and D. Let $\Psi_x = fX; \Psi_y = fY$ denote the sets of aggregate trade and investment done in equilibrium. This leads to Proposition 6:

**Proposition 6** If Conditions A and D hold, and there are increasing differences in investment, with positive (negative) externalities on efficient traders, $\Psi_x \subset M_x^{\alpha^1}$, $\Psi_y \subset M_y^{\alpha^1}$; and $\Psi_y \supset M_y^{\alpha^1}$.

**Proof:** See appendix.

This result is technically true even when there are decreasing differences in investment; although, as is shown in the proof, when there are decreasing differences the proposition is vacuous. That is, we cannot apply the strong-induced set order except in trivial cases. However, for decreasing differences we can once again derive a result comparable to our findings in the full-information case. Then we have the following:
Proposition 7  If Conditions A and D hold, and there are decreasing differences in investment, with positive (negative) externalities on efficient traders, \( \forall x \left[ M_x^{n-1} (0) M_x^n \right] \) and \( \forall y \left[ M_y^n (1) M_y^n \right] \).

Proof: See appendix.

This proposition may be viewed as complementary to Proposition 3, but there is a key difference between them (as well as between Propositions 2 and 6). Under public offers the principal’s decisions were inefficient due to the externalities consumed by nontraders; under private information inefficiency exists because of externalities on efficient traders. Furthermore, Propositions 6 and 7 are weaker than their full-information counterparts. For example, suppose \( E = \{ -1; 1; 2 \} \) and \( M^* = \{ 0; 1; 2 \} \). Then we can say \( E \left[ M^{n-1} M^n \right] \) but we cannot say \( E \left[ M^n \right] \). In general however, Proposition 6 does state that \( \sup E \cdot \sup M^n \).

Given that solutions to these programs are pairs \((x; y)\); one hopes for an analysis of how these pairs are distorted in tandem. Surprisingly, despite the different externalities at work essentially the same proposition describing how trade and investment move together holds in both information settings.

Proposition 8  Under Conditions D and A; suppose that there is an equilibrium \((\hat{x}, \hat{y})\) where aggregate trade \( X = \sum x \) is efficient. Then equilibrium investment \( \hat{y} \) is also efficient.

Proof: See appendix.

The intuition at work in the private offers setting seems like it should be different from the public offers setting. With public offers, investment does not directly affect externalities on nontraders and consequently distortions in investment must stem from distortions in aggregate trade. With private offers, externalities on efficient traders are what matter and investment does play a role here.

However, the intuition behind the proofs is the same in both cases. In both cases distortions are driven by the principal seeking rent from changes in trade. With public offers this is done by changing reservation utilities. With private offers this is done through the principal’s inability to commit to a public trade profile, but the principal can commit to investment. Thus, if the
the principal to distort investment whether or not offers are public or private. However, it could be the case that the same level of investment maximizes profits for various \( X \) and thus a distorted level of trade could be accompanied by an efficient investment choice.

5 The Role of the Firm

This section reconsiders the public offers game with a slight modification; we introduce a new actor, the firm, to the game. We will show that under some conditions the presence of the firm will counteract the distortions created by externalities. The intuition is that firms face different incentives than agents and are able to internalize externalities between agents when negotiating contracts.

The timing of the game is now as follows. In the first stage, each agent simultaneously decides whether or not to work for the firm; agents’ decisions are common knowledge. In the second round, the principal once again publicly commits to an investment decision and makes a publicly observable take-it-or-leave-it offer to each agent. In the third round, agents who do not work for the firm accept or reject their contracts and realize their payoffs as before.

If an agent agrees to work for the firm, then in the third round the firm decides whether or not to accept the agent’s contract. Regardless of whether or not the contract is accepted, the firm pays the agent a salary equal to the agent’s reservation utility plus a small amount \( \xi \). The firm keeps any additional surplus for itself.14

We will consider a weakened version of the principal’s preferred SPNE of this game, the principal’s weakly preferred SPNE. We define this type of equilibrium as follows.

Definition: A principal’s weakly preferred SPNE is any SPNE \( \pi \) such that there does not exist any other SPNE of the game which has both (a) the same \( \rho \)-round outcome as \( \pi \) and (b) a strictly greater payoff for the principal than \( \pi \).

\[14\] Thus, if it is efficient for agents to trade with the principal the firm will be able to make a positive profit.
This definition is weaker than the principal’s preferred SPNE because we are placing fewer restrictions on the first round. Obviously, any equilibrium which is a principal’s preferred SPNE is also a principal’s weakly preferred SPNE. We can still have multiple equilibria with the same first-round outcome under the principal’s weakly preferred SPNE; although it must be that in such a case the principal’s payoff is the same among these equilibria. We will assume that at an equilibrium exists. We again consider equilibria where all offers are accepted.

Of course, agents may not want to work for the rm in equilibrium, but what follows holds for situations where all, some, or none of the agents work for the rm.

What we will show is that as more agents work for the rm the principal’s decisions become in a certain sense “less distorted”. However, this may not be the case in general. The reason is that we have made almost no assumptions at all on \( u_i(0; x_i) \) and so obtaining monotone comparative statics with respect the number of agents working for the rm requires a little more machinery.

Condition M describes situations where as the size of the rm increases there will be a decrease in distortion:

\[
\text{CONDITION M: For all } i, x_i \geq 0; 1g \text{ and } u_i(0; x_i) = u(0; x_i): \]

Condition M (the Monotonicity condition) says that trade with agents is binary (i.e., it happens or it doesn’t happen) and that reservation utilities are a function of the sum of aggregate trade. This implies that externalities on nontraders are anonymous. Condition M is similar to Segal’s Condition S, but in fact our condition is more general and makes no assumptions about the symmetry of agent payoffs. Condition M implies condition D. Examples of research done under condition M include Katz and Shapiro (1986a) and (1986b), Kamien Oren and Tauman (1992), and Rasmusen, Ramseyer, and Wiley (1991).

As usual, we are interested here in comparing sets of solutions. Specifically, the following proposition compares first-best outcomes \( M^* \); the outcomes under the public offers game of Section 3, \( M \); and the outcomes under the presence of the rm which we denote \( M^F \): Let \( M^F_x = \bigcap x_i = x; (x; y) 2 M^F \) and \( M^F_y = \bigcap y (x; y) 2 M^F \):
Proposition 9 Under conditions A and M, the presence of the firm will lessen the distortions caused by externalities. Specifically,

(A) If there are increasing differences between trade and investment and there are
   a) positive externalities, then $M_x > M^F_x$ and $M_y > M^F_y$; and
   b) negative externalities, then $M_x < M^F_x$ and $M_y < M^F_y$;

(B) If there are decreasing differences between trade and investment and there are
   a) positive externalities, then $M_x < M^F_x$ and $M_y > M^F_y$; and
   b) negative externalities, then $M_x > M^F_x$ and $M_y < M^F_y$;

(C) As more agents are employed by the firm, equilibrium trade and investment move closer to their first-best levels. That is, suppose there are two possible sets of agents working for the firm in equilibrium denoted $F^0$ and $F$ where $F^0, F$; and denote their respective equilibrium outcomes $M^F_0$ and $M^F$: If externalities are positive and there are increasing differences we have $M_x > M^F_x$ and $M_y > M^F_y$ and similarly for all other cases.

Proof: see appendix.

This paper’s title suggests that we are interested in the relationship between externalities, investments and the boundaries of the firm. Looking at proposition 9, we see that in our model there is no relationship. This proposition suggests that the existence of firms will abrogate the distortions created by externalities. Importantly, the proposition holds even when $\bar{A} = f0$ and there is no investment. Thus, firms can lead to efficiency gains even when there is no investment and no hold-up problem.

This finding differs significantly from the standard theories on the boundaries of the firm, which emphasize how asset acquisition leads to efficient investment; these stories generally assume that the party being acquired or the party doing the acquisition also makes an investment decision. While there is no doubt that the hold-up problem is important to understanding the boundaries of the firm, this paper shows that firms can do much more than just ensuring efficient investment in markets with incomplete contracts.

Here, the firm pays workers their reservation wages no matter what and consequently the firm views reservation wages as sunk costs when considering a contract. This means that when the principal offers contracts to agents working for the firm she has no reason to deviate from first-best outcomes; she can no longer obtain rents from distorting their reservation utilities. This suggests that if all agents work for the firm, the principal’s problem is identical to the first-best...
problem and the outcomes will be \textit{first} best.

6 Conclusions

This paper begins an analysis of how the presence of multilateral externalities affects investment decisions by generalizing a model of bilateral negotiation presented in Segal (1999). Using a general model applicable to a number of different situations, we have derived sufficient conditions for making robust predictions concerning the effects of externalities. Under full information, inefficiency arises because of the effects externalities have on nontraders. Under private offers, distortions occur because of the externalities consumed by agents who are trading efficiently. The difference in the type of externalities which create distortions suggests that outcomes under private offers may be efficient while outcomes under full information may not.

Additionally, we have shown that it is not only the nature of externalities, but also the complementarity between investment and trade that determines the distortions on investment. These distortions occur despite the facts that investment does not directly affect outside opportunities, that there are no issues of asset allocation, and that there is no uncertainty regarding the profitability of exchange for a given trade profile.

Finally, we have shown that the existence of firms will allow for the internalization of these externalities and this may increase efficiency in the market. This is true even when there is no hold-up problem. This finding could have significant empirical implications for industrial organization research. Hart (1995) points out that there really is no evidence for the property-rights approach to the firm that goes beyond impressionistic. However, it could be the case that in some situations the property-rights theory of the firm makes different predictions than a theory of the firm based on externalities.

For example, conglomerate mergers are usually considered at odds with the property-rights theory of the firm but could be explained by theories based on externalities. Moreover, the analysis of the firm presented in this paper shows that the firm can counteract distortions even when the
parties working for the .rm don't invest, a prediction very much at odds with the standard theory. In any event, the fact that the traditional theory of the .rm now has a little competition makes it possible to compare different theories and this can only improve our understanding of the boundaries of the .rm.

There are a number of limitations to the present analysis, however. First, our analysis only considers one-sided investment opportunities; agents are not able to invest. Further, the results on the role of the .rm should be taken as suggestive and they demand further research. Theoretically, it would be important to know how successfully these results can be generalized in various negotiation settings. Additionally, future research could consider games where contracting is not bilateral (such as auctions). Finally, a study of robust predictions concerning the interaction between externalities and other factors affecting investment could yield new insights in this setting. In the absence of these alternate influences, however, this paper establishes that a meaningful analysis of investment and externalities is possible.
7 Appendix

Note: Propositions 2, 3 and 9 here rely on Topkis' (1998) Theorem 2.8.1. To facilitate comparison between this paper and Segal's work and for ease of exposition for Propositions 3 and 6, this paper uses the strong induced set order while Topkis results use the induced set order. In the interest of assuaging the skeptical reader Topkis' theorem is included here, as well as a lemma showing that the theorem is sufficient for proving our propositions. Further, a proof of supermodularity for each proposition is carried out in detail. All of these results may be omitted in the final version of this paper.

PROOF OF LEMMA 1: (this is similar to the proof in Segal): Consider the case of positive externalities on nontraders. Take some \( X^0 \); \( X \) \( 2 \) \( A \) where \( X^0 \); \( X \): Suppose \( x \) \( 2 \) \( A \) such that \( P_i x_i = X \) and \( R(X) = P_i u_i(0; x_i) \): By condition D there exists \( X^0 \) \( 2 \) \( A \) such that \( x^0 \); \( x \) and \( P_i x_i = X^0 \): With positive externalities on nontraders, this implies that \( R(X^0) \cdot P_i u_i(0; x_i^0) \cdot P_i u_i(0; x_i) = R(X) \) wherever \( R(X^0) \) is defined. A similar argument holds for negative externalities.

PROOF OF PROPOSITION 2: Consider the parameterized program \( \max_{X \in A; y} A(X; y) \mid z R(X) \); where \( z = 0 \) corresponds to the surplus-maximization program and \( z = 1 \) corresponds to the principal's profit-maximization program. Let us consider the case of positive externalities. We will use the following definitions and theorem:

Definition: Suppose \( f \) is a real-valued function on a lattice \( X \). If \( f(x_0) + f(x^0) - f(x_0 \wedge x^0) \) for all \( x_0; x^0 \) in \( X \), then \( f(x) \) is supermodular on \( X \).

Definition: Suppose \( X \) is a lattice with ordering relation \( \cdot \). Then the induced set ordering \( \cdot \) on \( X \) is such that \( X^0 \cdot X^0 \) in \( P(X) \) if \( x^0 \cdot x^0 \) and \( x^0 \cdot x^0 \) imply that \( x^0 \cdot x^0 \cdot x^0 \cdot x^0 \) and \( x^0 \cdot x^0 \cdot x^0 \cdot x^0 \), where \( P(X) \) is the power set.

Topkis (1998) Theorem 2.8.2: If \( X \) and \( T \) are lattices, \( S \) is a sublattice of \( X \cap T \); \( S \) is the section of \( S \) at \( t \) in \( T \), and \( f(x, t) \) is supermodular in \( (x, t) \) on \( S \), then \( \arg \max_{x \in S} f(x; t) \) is
increasing in \( t \) on \( \mathbb{R}^2 \); \( \arg \max_{x \geq s_t} f(x; t) \) is nonempty.

On page 74 Topkis notes that “increasing” here is meant in the sense of induced-set ordering. The following lemma shows that this suffices for proofs involving the strong induced set ordering used in our propositions.

Lemma A.1: Let \( X; X^0 \) be lattices whose elements are taken from compact subsets of \( \mathbb{R}^2 \); denote their elements \( (x_1; x_2); (x_0^1; x_0^2) \) respectively. Let \( X_1 = f(x; z) \) \( \forall X \) for some \( z \); \( X_2 = f(x; z; x) \) \( \forall X \) for some \( z \); and similarly for \( X_1^0 \) and \( X_2^0 \). If \( X \) \( \forall X \) then \( X_1 \) \( \forall X \) and \( X_2 \) \( \forall X \).

**Proof of Lemma A.1:** Suppose instead that \( X_1 \geq X_1^0 \) and \( (x_0^1; x_0^2) \) \( \forall X \) were \( x_1 \), \( x_0^1 \) but either \( x_1 \geq X_1^0 \) or \( x_0^1 \geq X_1 \) (or both). In either situation it cannot be the case that \( x_1 \geq X_0^0 \) \( \forall X \) and \( x \sim x^0 \) \( \forall X \), but these are true by the definition of \( \forall X \) and \( \forall X \) are thus true by assumption. So we have a contradiction and \( X_1 \) \( \forall X \): Identical logic with \( X_2 \) and \( X_2^0 \) completes the proof of Lemma A.1.

Given that our choice-variable space is composed of lattices and by assumption the \( \arg \max \) is not empty, by Lemma A.1 we know that Topkis’ (1998) Theorem 2.8.2 will suffice for proving Proposition 2; we will prove that the objective function is supermodular in \( (i X; i y; z) \):\(^{15}\). Suppose we compare two sets \( (X^1; Y^1; Z^1); i = a, b \). We wish to show that \( A(X^a; y^a) + A(X^b; y^b) \) \( \forall X \) and \( A(X^a; y^a) \) \( \forall X \) and \( A(X^b; y^b) \) \( \forall X \). Then \( \forall X \) there are four cases to consider:

**Case 1** \( X^a = X^b = y^a = y^b \): Then the result is trivial.

**Case 2** \( X^a = X^b \neq y^a \neq y^b \); and \( z^a \neq z^b \): Then we need only worry about the \( i z \mathbb{R}(X) \) part of the function; the rest drops out. By Lemma 1 we know that \( z^a[R(X^a) \mathbb{R} R(X^b)] \cdot z^a[R(X^a) \mathbb{R} R(X^b)] \) and rearranging gives \( i z^a[R(X^a)] \cdot z^b[R(X^b)] \cdot i z^b[R(X^a)] \cdot z^a[R(X^b)] \) and the result holds.

**Case 3** \( X^b = X^a = y^a = y^b \); and \( z^a \neq z^b \): Then we need only worry about \( A(.) \). Our

\(^{15}\) We say \( f \) is supermodular on \( \mu = (i \ a; b) \) if \( \mu = a^1 \mu = a^0 \cdot b^0 \).
assumption of increasing differences implies that $A(X^a, y^a) \leq A(X^b, y^b)$ and we can rearrange this to get $A(X^a, y^a) + A(X^b, y^b) \geq A(X^a, y^a) + A(X^b, y^b)$; and the result holds.

Case 4) $X^a \geq X^b y^b, y^a$; and $z^a, z^b$. Now nothing drops out. However, applying the same steps from Cases 2 and 3 again yields the result.

Since the function is supermodular in $(X; y; z)$ the result follows from Topkis’ (1998) Theorem 2.8.2. For the proof of negative externalities one must show that the objective function is supermodular in $(X; y; z)$; the proof is similar (and in fact easier).¥

PROOF OF PROPOSITION 3: The logic here follows that for Proposition 2. We will consider the case with negative externalities and we claim that the objective function is supermodular in $(X; y; z)$: We consider four cases.

Case 1) $X^a, X^b y^b, y^a$; and $z^a, z^b$: Then the result is trivial.

Case 2) $X^b, X^a y^a, y^b$; and $z^a, z^b$: Then the $A(.)$ function drops out and we simply note that $z^b[R(X^a) \setminus R(X^b)] < z^a[R(X^a) \setminus R(X^b)]$ by Lemma 1 and the result holds.

Case 3) $X^a, X^b y^a, y^b$; and $z^a, z^b$: Now the $R$ function drops out. By the assumption of decreasing differences we have $A(X^a, y^b) \leq A(X^b, y^b) > A(X^a, y^a) \geq A(X^b, y^a)$; or $A(X^a, y^b) < A(X^b, y^a) + A(X^a, y^b)$; and the result holds.

Case 4) $X^a, X^b y^a, y^b$; and $z^b, z^a$: Now nothing drops out, but once again by appealing to the two above cases we can show that the condition holds.

Since the function is supermodular in $(X; y; z)$; the result follows from Topkis’ (1998) Theorem 2.8.2. The proof using the fact that with positive externalities the objective function is supermodular in $(-X; y; z)$ is similar.¥

PROOF OF PROPOSITION 4: Suppose to the contrary that there is an equilibrium profile $(x; y)$ where aggregate trade is efficient (and so $X \geq M_x$) but investment $y$ is distorted. Then for any $y^a \leq M^a_x$ such that $(x; y^a) \geq M^a_x$ we have $A(X; y) \leq R(X), A(X; y^a) \leq R(X) or A(X; y)$,
A(\(X; y^n\)), but this inequality must bind by the definition of \(M^n\). This implies that \(y \not\in M^n_y\) which is a contradiction.

\textbf{PROOF OF PROPOSITION 5:} For any \((\hat{x}, \hat{y}) \not\in \Psi\); condition (6) shows that
\[
X \sup_i u_i(\hat{x}_i; x_i; y_i) = f(x)_i g(y)_i = X \sup_i u_i(\hat{x}_i; x_i; y_i) = f(x)_i g(y)_i
\]
and \((\hat{x}, \hat{y}) \not\in M^n; \Psi\).

\textbf{PROOF OF PROPOSITION 6:} We consider the case of negative externalities on efficient traders. Suppose that \((\hat{x}, \hat{y}) \not\in M^n\) and \(\hat{x} \not\in \Psi_x \[ M^n_x \) and \(\hat{y} \not\in \Psi_y \[ M^n_y \) and \((\hat{x}, \hat{y}) \not\in \Psi\) with
\[
P_i x_i = \hat{x}_i - X^n \sup_i \hat{x}_i \quad \text{and} \quad \hat{y} = y^n; \]  \[16\] Notice that this assumption does not hold except in trivial situations in the case of decreasing differences. To see this, suppose that there were decreasing differences. Then by assumption 0, \(A(X^n; \hat{y}) \geq A(X^n; y^n)\); and decreasing differences imply \(A(X^n; \hat{y}) \geq A(X^n; y^n)\), \(A(\hat{x}; \hat{y}) \geq A(\hat{x}; y^n)\); where the terms on the right-hand side would be the payoffs the seller could get at various levels of investment given equilibrium beliefs and the equilibrium trade profile. However, these two equalities imply \(A(\hat{x}; \hat{y}) = A(X^n; y^n)\) but this could never be strict by our assumption that \(\hat{y}\) is an equilibrium level of investment.

To complete the proof, we first note that trivially \(X^n \not\in \Psi_x \[ M^n_x \) and \(y^n \not\in \Psi_y \[ M^n_y \) and all we have to show is \(\hat{x} \not\in M^n_x \) and \(\hat{y} \not\in M^n_y\). By Condition D there exists \(x^*\) such that \(X^n = P_i x_i^n\) and \(x^n = \hat{x}\). Then we can write
\[
A(\hat{x}; \hat{y}) = X \sup_i u_i(x_i; \hat{x}_i; y^n) = f(x)_i g(y)_i = X \sup_i u_i(x_i; \hat{x}_i; y^n) = f(x)_i g(y)_i = A(\hat{x}; \hat{y}) = A(X^n; y^n).
\]
The first inequality comes from condition (6), the second from our assumption that externalities are negative, and the last equality from Condition A. Thus, \(\hat{x} \not\in M^n_x \) and \(\hat{y} \not\in M^n_y\) and the condition holds. The case for positive externalities on efficient traders is similar.

\[16\] If we didn't assume \((\hat{x}, \hat{y}) \not\in \Psi\) we could do \(x\) and \(y\) one at a time, by selecting first \(x\)'s pairwise component in \(\Psi\); and later \(y\)'s. This is thus a simplification and done wlog.
PROOF OF PROPOSITION 7: In fact, this proof is very similar to the proof for Proposition 6. We again consider the case of negative externalities on efficient traders. Suppose that \((x^*;y^*) \in M_x^n\) and \(\hat{y} \in M_y^n\); with \(P_i X_i = X^* \cdot X^n\) and \(\hat{y} \cdot y^n\). Notice that this assumption does not hold except in trivial situations in the case of increasing differences. For simplicity assume \((\hat{x},\hat{y}) \in X\); that is, the pair \((\hat{x},\hat{y})\) is a solution to (6).

To complete the proof, we first note that trivially \(X \in X\); so all that is left to show is that \(\hat{x} \in M_x^n\) and \(\hat{y} \in M_y^n\): By Condition D there exists \(x^*\) such that \(X = P_i x_i^n\) and \(x^* \cdot X\); Then we can write

\[
A(X; y) = \sum_{i} u_i(x; x_i; y) - f(x) - g(y) = \sum_{i} u_i(x_i^n; x_i; y^n) - f(x^n) - g(y^n) = A(X^n; y^n),
\]

The first inequality comes from condition (6), the second from our assumption that externalities are negative, and the last equality from Condition A. Thus, \(\hat{x} \in M_x^n\) and \(\hat{y} \in M_y^n\) and the condition holds. The case for positive externalities on efficient traders is similar.

PROOF OF PROPOSITION 8: Suppose to the contrary that there is an equilibrium profile \((\hat{x}; \hat{y})\) where aggregate trade is efficient (and so \(\hat{x} \in M_x^n\)) but investment \(\hat{y}\) is distorted. Then for any \(y^* \in M_y^n\) such that \((\hat{x}; y^*) \in M^n\) we may write

\[
A(X; y^*) = \sum_{i} u_i(x; x_i; y^*) - f(x) - g(y^*) = \sum_{i} u_i(x_i^n; x_i; y^n) - f(x) - g(y^n) = A(X^n; y^n),
\]

where the right hand side expression is efficient and thus by definition the inequality binds. But this implies that \((\hat{x}; \hat{y})\) is first best which is a contradiction.

PROOF OF PROPOSITION 9: We proceed by backwards induction. In the final round agents who do not work for the firm face the individual rationality constraint from before:

\[
u_i(x; y) \geq t_i,
\]

\(u_i(0; x_i) \in 2W\) where \(W\) is the set of agents who work for themselves and
not the rm.\textsuperscript{17} For agents who work for the rm, the rm decides whether or not to accept the contract. Since the rm pays the reservation utility $u_i(0; x_i)$ to agent $i$ no matter what this drops out of the rationality constraint: $u_i(x; y) \mid t_i, 0 \in \mathbb{N}$.

In the second round the principal’s problem is the same except that the principal faces two different types of constraint; the principal takes the set $W$ as given. Once more all constraints will bind in equilibrium. In the rst round agents decide whether or not to work for the rm. Characterizing this decision process is not possible without further restrictions on the game but fortunately the rst round is superuous when proving proposition 9.

The principal’s problem is now $\max_{X} \forall Y \in \mathbb{A} A(X; Y) \mid R(X; W)$ where

$$R(X; W) = \min_{x \in \mathbb{A}} \min_{i \in W} u_i(0; X; x_i) : x = X$$ \textsuperscript{18}

Note that Lemma 1 still applies to $R(X; W)$ for any given set $W$: The following additional lemma will be needed:

**Lemma A 2**: Under condition S, if externalities on nontraders are positive (negative), $R(X; W) \mid R(X^0; W)$ is nondecreasing (nonincreasing) in $W$ for all $X$, $X^0$ and $W$.

**Proof of Lemma A 2**: Suppose we add $k > 0$ new elements to the set $W$: Let us consider any one of the new agents now self employed; call him agent $j$: Suppose externalities on nontraders are positive. Then by Condition M $R(X; W)$ will increase by at least $u_j(0; X; x_i)$ and $R(X^0; W)$ will increase at most by $u_j(0; X^0)$: However, $u_j(0; X; x_i)$, $u_j(0; X^0)$ since $X < X^0$ by Condition M and since externalities are positive. Thus $R(X; W)$; $R(X^0; W)$ weakly increases after the addition of agent $j$ to $W$: This is true for any $j$ and so $R(X; W)$; $R(X^0; W)$ is nondecreasing in $W$.

If externalities on nontraders are negative, then $R(X; W)$ will increase by at most $u_j(0; X; x_i)$ and $R(X^0; W)$ will increase by at least $u_j(0; X^0)$: But $u_j(0; X; x_i)$, $u_j(0; X^0)$ since $X < X^0$ by

\textsuperscript{17} The set $W$ is thus taken from the power-set of $\mathbb{N}$ denoted $P(\mathbb{N})$: Any such set $P(\mathbb{N})$ is a lattice with ordering relation $\mu$ where the join of two subsets is their union and the meet of two subsets is their intersection.

\textsuperscript{18} If $W = \emptyset$ then $R(X; W) = 0$: In this case the principal’s problem is the same as the problem of maximizing surplus and thus $M = M^*$. 30
Condition M and since externalities are negative. This is true for any $j$ and so $R(X; W) \leq R(X^q; W)$ is nonincreasing in $W$: This completes the proof of Lemma A2.

The seller's objective function is once more maximized over a lattice and so we again rely on Topkis' Theorem 2.8.2. This Theorem will actually prove parts A, B, and C of the proposition. Consider the case of positive externalities and increasing differences. We will prove that the principal's objective function is supermodular in $(X; y; W)$.

There are four cases to consider:

Case 1) $X^a \leq X^b; y^a \leq y^b; \text{and } W^b \leq W^a$: Then the result is trivial.

Case 2) $X^a \leq X^b; y^b \leq y^a; \text{and } W^b \leq W^a$: Then we need only worry about the $A(X; y)$ part, the rest drops out. This result follows from the proof of Proposition 2, case 3.

Case 3) $X^a \leq X^b; y^a \leq y^b; \text{and } W^a \leq W^b$: Then we need only worry about the $R(X; W)$ part. By Lemma A2, $R(X^a; W^b) \leq R(X^b; W^b) \leq R(X^a; W^a) \leq R(X^b; W^a)$ and rearranging gives $R(X^a; W^a) R(X^b; W^b) R(X^a; W^b) R(X^b; W^a)$ and the result holds.

Case 4) $X^a \leq X^b; y^a \leq y^b; \text{and } W^a \leq W^b$: Then nothing drops out but by Cases 2 and 3 it is easy to show the result holds.

Thus the objective program is supermodular in $(X; y; W)$: To complete the proof one must show that the objective function is supermodular in $(X; y; W)$ for negative externalities and increasing differences, in $(X; y^a; W)$ for positive externalities and decreasing differences, and in $(X; y^a; W)$ for negative externalities and decreasing differences. All of these cases are similar.
8 References


Kamien, Morton, and Tauman Yair, “Fees Versus Royalties and the Private Value of a Patient,”


