Public Goods, Hidden Income, and Tax Evasion:

Some Nonstandard Results from the Warm-Glow Model

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April 2014

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Abstract

A large literature explores crowd out in situations where public goods are jointly provided; work in this area typically depicts a tax system where individuals take taxes as given. But in some settings, such as those in developing economies, efforts to evade or avoid taxes may be widespread. Using the canonical warm-glow model, this paper considers joint pubic-good provision in a setting where individuals can evade taxes by hiding their income. The model's implications change significantly in this setting: with hidden income, stronger warm glow will lead to greater crowd out, not less. Using research on crowd out and inter-family transfers, I present suggestive evidence that the model's results may help to reconcile divergent estimates of crowd out in the literature.

*I am grateful to Tom Gresik, Terry Johnson, Kali Rath, Guillermo Roque, Tihtina Zenebe Gebre, and to audiences at George Mason University, McMaster University, the University of California-San Diego, Peking University, and the University of Notre Dame for comments and assistance. Email the author at dhungerm@nd.edu

Introduction

The joint provision of public goods is familiar in both developed and developing societies. One of the most studied aspects of this activity is the potential for crowd out: efforts by one party to increase provision of the good may reduce, or crowd out, provision from others. Researchers are frequently interested in whether crowd out leads to less effective government intervention; models of crowd out thus typically include a government using tax revenue to provide some of the public good. Traditionally, individuals in crowd out studies are assumed to take their tax burdens as given. But, in reality, efforts to hide resources from taxation are commonplace in many developing economies (and elsewhere). This paper thus asks: does the ability to hide income alter the standard crowd-out story in important ways? Using the canonical model of crowd out in the literature, the analysis here suggests that it does.

The study here focuses on the warm-glow model of crowd out; individuals in the warm-glow model view resources that they voluntarily donate to the public good as distinct from tax-based contributions (i.e., individuals get a "warm glow" from voluntary donations). The implications of warm glow for crowd out are well known: stronger warm glow leads to less crowd out and more effective policy intervention. Intuitively, if the government (or perhaps a Non-Governmental Organization or NGO), intervenes to provide more of a public good, warm glow causes individuals to view this intervention as a poor substitute for their own donations. The reluctance to substitute government funds with private donations reduces crowd out.

I show that, if individuals can lower their taxes by choosing to hide a portion of their income from the government, this relationship between warm glow and crowd out is reversed: stronger warm glow will correspond with more crowd out, not less. Intuitively, hiding income is an action that is beneficial to the individual but socially costly. As income is hidden, tax revenues fall and the level of the public good falls. An exogenous intervention to increase the public good can thus induce a response—more hidden income—that makes the intervention less effective. This hidden-income response will be large when individuals place a high value on their own voluntary donations relative to the value they place on the public good. But this is what the warm glow captures: stronger warm glow means individuals have stronger preferences for their own private behavior relative to social outcomes, and thus stronger warm glow will exacerbate a hidden-income-response that negates policy intervention. I illustrate this result with a simple example and show that the presence of hidden income can lead to much larger crowd out than one would see without hidden income. The analysis here assumes interiority (as is standard) and that individuals choose to hide some, but not all, income. In Section 3, I discuss these assumptions and the extension of this result to situations where some individuals hide income and some do not.

Next, I compare the effect on public-good provision from a change in the tax rate to the effect from a

change in the cost of hiding income. While Andreoni (1990) found that subsidies have stronger effects than taxation on public good provision, the relative merit of taxation with hidden income is ambiguous. Further, in this setting the relative efficacy of taxation on the level of the public good may depend on the price sensitivity of demand for the public good, an unconventional warm-glow result but a possibility considered by earlier work (cf. Feldstein, 1980). Finally, I show that an exogenous increase in public good provision (such as an increase in NGO provision) can lower tax revenue, and that this effect is greater as warm glow increases.

Many prior crowd out studies consider settings where institutions are relatively strong and efforts to hide income appear to be limited (Schneider and Enste, 2000); the results of this paper indicate that in settings where efforts to hide income are pervasive, crowd out may behave quite differently. For instance, when looking at the decisions of adult children in a family to transfer money to their elderly parents, one might wonder whether the introduction of a government pension program for the elderly would crowd out such inter-family transfers. In this case, children may make contributions for both altruistic reasons (concern for their parents' well-being) and for warm-glow-related reasons (e.g., a child's contributions confer parental approval). The traditional analysis would suggest that stronger warm glow incentives would lead to less crowd out and a greater net transfer of income to the elderly from the pension. The analysis here would suggest that, if efforts to hide income are widespread, stronger warm glow would instead increase crowd out.

I undertake a suggestive exploration of this possibility by examining how estimates in the development literature on crowd out and inter-family transfers correlate with warm glow. My measure of warm glow is based on a series of questions in the World Values Survey. Consistent with the standard model, I find that, in countries where individuals have relatively low self-stated inclinations to hide income, published crowd out estimates are positively related to warm glow. But consistent with the model here, in countries with high inclinations to hide income this relationship turns negative. While suggestive, these results help to reconcile the highly diverse set of crowd out estimates extant in the development literature

These results also contribute to the small area of work that explores how crowd out varies in different circumstances. Payne (2009) suggests that the setting where crowd out occurs may have a large influence on the magnitude of crowd out, but the body of work relating environmental attributes to crowd out behavior is limited and has not considered how efforts to avoid taxation—or features of the tax system more generally—may impact the efficacy of policy interventions.[1] The next section provides motivation for the analysis and

[1] Attributes considered by prior work include community size (Ribar and Wilhelm, 2002), community diversity (Hungerman, 2009), and income (Cox, Hansen, and Jimenez, 2004), but empirical estimates seem to vary in cases where these factors seem unimportant (cf. Kingma, 1989; Straub and Manzoor, 2005; Payne, 1998; Khanna and Sandler, 2000; Cox, Hansen, Jimenez, 2004; Gibson, Olivia, and Rozelle, 2011). Some work has also explored variation in crowd out by considering variation in preferences, rather than in technologies. Among such papers, perhaps the paper closest to this one is a study by Krause (2011), who considers a model where utility depends upon one's voluntary giving relative to the giving of others; Krause provides numerical examples where stronger preferences for "out-donating" others leads to greater crowd out. While interesting, this

introduces the model. Section 3 presents the analysis. Section 4 considers prior crowd out estimates, and section 5 concludes.

2. The Warm-Glow Model with Hidden Income

2A. The Potential Role for Hidden Income

This section presents the basic warm glow model with hidden income. Before considering the model, however, it will be useful to discuss the significance of hidden income as an economic activity. First, one might ask, do people hide income? The answer frequently appears to be "yes." Recent work has documented numerous settings where individuals avoid tax obligations by hiding or concealing taxable resources. Schneider and Enste (2000) show that "shadow" or "clandestine" economic activity is common in developed countries, and they argue that efforts to conceal income may be even more common in developing societies; Andreoni, Erard, and Feinstein (1998) also discuss high levels of tax evasion in developing countries.[2]

Further, there is evidence that decisions to conceal income may respond to pressures to provide resources that will be transferred to others. For example, Schneider and Enste (2000) conclude that the rise of social security burden (a source of crowd out that will receive more attention below) is "one of the most important causes" of underground economic activity in the world. Social security programs are a case where those making interpersonal transfers (e.g., adult children supporting the elderly) may thus do so through a public program (by complying with taxation) or not. A related decision could be whether to earn income for remittances through the formal or informal sector. As discussed below, the model here extends to individuals deciding how to divide time between formal and informal labor; work suggests that both formal and informal labor opportunities matter for rural-to-urban migrant workers (Meng, 2001; Banerjee, 1983), sometimes within a given household (Merrick, 1976). Such a decision may also involve a warm glow component: a government social security or pension program may be viewed as a non-perfect substitute for family remittances, e.g., because supporting one's parents directly provides warm glow by securing familial approval.[3] Additionally, observers have noted that informal activities may hamper or discourage cooperation with government programs to help those in need (Kaser, 2000; Foster, 1985; Monaco-Mancini, 1999).

The discussion below focuses on a situation where individuals choose to pay taxes on a portion of their resources; but in many settings in developing societies individuals may either be outside of or inside of the

notion of "relative giving" differs from the traditional depiction of warm glow considered here.

[2]Gordon and Li (2005) also argue that difficulty in monitoring taxable activity is a salient aspect of economic policy in developing countries. While developing countries often rely less than other countries on individual-income taxation (which is the type of tax considered here), Miller et al. (2011) show income taxation is common in both poorer and richer nations.

[3]It has been recognized at least since Warr (1982) that such interpersonal transfers can be considered contributions to a public good.

formal economy entirely. There are several observations on this point. First, importantly, the analysis here can incorporate situations where some individuals when choosing how much income to hide arrive at "corner solutions." Second, there is evidence that, at least in some settings in developing economies, the decision to pay some taxes but to underpay them is economically relevant. For example, Alm, Bahl, and Murray (1991) use a variety of data from Jamaica and find that lost taxes from underreporting results in large declines in tax revenue while they also conclude that tax evasion from nonfiling is "enormous."[4] As mentioned above, the model can also extend to cases where individuals choose to divide labor between informal and formal employment. The model here can also extend to settings where efforts to raise revenues from public goods is informal. For example, a household might face a request to provide an interhousehold transfer (such as in-kind aid or financial assistance to a distressed household) through a community organization; if the household has an incentive to provide such aid directly (for instance, direct aid might help the household fulfill a familial obligation or improve the household's reputational status)[5] such incentives might lower the household's professed ability to participate in the community organization.[6] Ultimately, the relevance of these applications of the model is an empirical question; it is encouraging that the suggestive results below indicate the analysis here may have empirical relevance for reconciling past research in the area of remittances.

Thus there is evidence that individuals may hide income from the government or each other, that hidden income may play a role in interhousehold transfers, that the decision to hide resources is responsive to pressure to share income, and that efforts to hide income involve costly behavior. The model described next considers how such efforts to hide income could impact crowd out.

[4]Burgess and Stern (1993) also discuss underreporting taxes (see section 5.2 of their paper); Alexandrova (2013) gives a non-academic discussion. Moreover, even when individuals comply with the tax code, the model here can also extend to situations of tax avoidance, where individuals take legal but inefficient actions to avoid taxation; Alm, Bahl, and Murray (1991) find evidence that reliance on tax-favored forms of compensation leads to a large discrepancy between potential and realized tax revenue (see Table 6 in their paper).

[5]Shokkaert (2006) writes that several incentives fitting the notion of warm glow would appear to be "essential" in the promotion of household transfers; although he notes that non-altruistic motives for such transfers appear to be important and that more work is needed to refine the exact determinants of such behavior.

[6]Olken and Singhal (2011) present evidence that, after including "informal" forms of taxation, households in developing countries may spend a reasonably large share of income on taxes; the average household in the modal country they study spends about 8% of income on informal and formal taxes (see Table 4 in their paper; the 8 percent figure was arrived at by dividing the values in row 1 by those in row 3). Their results exclude some contributions to organizations such as local social organizations; including such activity might push the relevant concept of expenditures higher. Recent work has also documented efforts to hide income in these settings; Jakiela and Ozier (2011), Dupas and Robinson (2013), and Goldberg (2010) all argue that individuals in a developing economy may hide income in an effort to avoid pressure from others in their community to share (see also Kinnan, 2012). These papers also highlight potential costs from these behaviors; for example, Jakiela and Ozier (2011) report results from rural Kenya where women choose to hide income earned during an experiment despite the fact that doing so reduces expected earnings, and Dupas and Robinson (2013) show that women in Kenya are willing to invest in savings accounts with negative de facto interest rates in an effort to conceal their income from others.

2B. Introducing the Basic Model

The model is a standard warm-glow model with two alterations. First, I consider a case where taxes are proportional to income rather than lump sum. This change is, in itself, unimportant and would not alter the standard relationship between warm glow and crowd out. Second, I allow individuals to hide income from taxation. Consider a model where individuals and a central authority both contribute towards a public good. Let n be the total number of individuals. Individual i receives utility from three things: consumption of a private good, c_i, the total amount of the public good provided in the community, Y, and a "warm glow" component that depends upon the individual's own voluntary public-good donation, g_i. The individual's preferences may be expressed by the utility function $U^i(c_i, Y, g_i)$, which is continuous, twice differentiable and quasi-concave. It is assumed that goods are normal.

Individual i is endowed with income w_i. Income is subject to taxation at tax rate τ, $1 > \tau > 0$. However, individuals may choose to hide an amount s_i of their income; individuals do not pay taxes on hidden income. Individuals may use hidden income (along with net-of-tax unhidden income) to consume the private good or to make donations that provide warm glow. The "tax" levied here could refer to income taxes levied by the government, in which case efforts to hide income would reflect efforts to avoid income taxes. Alternately, the tax here could reflect non-governmental efforts (such as efforts from an NGO or community organization) in which case efforts to hide income may reflect actions to conceal resources from community members, NGO officials, or family members.

Hiding income is costly; a fraction θ of hidden income is lost. This lost income may reflect, for example, an expected loss which depends upon the possibility of an audit or resources exerted in the act of hiding. As mentioned in prior subsection, there is evidence that individuals are willing to incur such costs when hiding income from others. More generally, the loss of income represents the fact that efforts to avoid taxation can create deadweight loss.[7] It is assumed that the cost of hiding resources is small enough to make hiding an attractive option, $\theta < \tau$, or else individuals would never choose to hide anything. A situation where the marginal cost of hiding is non-constant is discussed in the appendix. An individual's budget constraint is:

$$c_i + g_i = (w_i - s_i)(1 - \tau) + s_i(1 - \theta).$$

The first term to the right of the equal sign represents "unhidden" income, net of taxes. The second term represents untaxed hidden income, net of deadweight loss. Together they equal *disposable income*.[8]

[7] In addition to the evidence in the previous subsection, see (e.g.) Slemrod (2007) for a discussion of the efficiency costs of tax evasion, as well as some interesting examples of the efforts individuals have taken throughout history to hide their resources from taxation.

[8] This budget constraint could also represent a decision to split time between formal and informal labor. If total time working was T, informal labor was s, formal labor earned wage w_f, and informal labor earned a premium α over net-of-tax formal labor,

As is standard, let the public good be provided by a simple linear technology that aggregates all donations and tax revenue: $Y = \sum_k (w_k - s_k)\tau + g_k$. For any individual i this can be written as $Y = Y_{-i} + (w_i - s_i)\tau + g_i$, where $Y_{-i} = \sum_{k \neq i}(w_k - s_k)\tau + g_k$. Individuals will choose optimal levels of c_i, g_i, and s_i while taking Y_{-i} as given. Re-arranging the above, let $g_i = Y - Y_{-i} - (w_i - s_i)\tau$. Plugging this into the budget constraint yields $c_i = w_i - s_i\theta + Y_{-i} - Y$. This expression resembles the standard warm-glow depiction of private consumption with the additional term $s_i\theta$; this new term appears because the decision to hide income affects the total amount of resources available in the community.

Substituting in for c_i and g_i, the individual's maximization problem can be represented as involving the optimal choices of Y and s_i :

$$\max_{Y,\, s_i} \; U^i \left(w_i - s_i\theta + Y_{-i} - Y,\; Y,\; Y - Y_{-i} - (w - s_i)\tau\right). \tag{1}$$

The first-order condition with respect to Y can be written as:

$$U^i_Y + U^i_{g_i} = U^i_{c_i} \tag{2}$$

Equation (2) is standard and shows that the marginal utility from private consumption must equal the marginal utility (both from the public good and the warm glow) from a voluntary donation. The first order condition with respect to s_i is $-\theta U^i_{c_i} + \tau U^i_{g_i} = 0$. Using (2) to plug in for $U^i_{c_i}$, this can be written:

$$\frac{U^i_Y}{U^i_{g_i}} = \frac{\tau - \theta}{\theta} \tag{3}$$

Equation (3) reflects incentives to hide income and merits discussion; it captures much of the intuition for the analysis.

The first-order condition in (3) equates a marginal rate of substitution with a ratio of prices, where the relevant "price" is not the price of a commodity but rather the cost of hiding income. Suppose an individual decides to hide a dollar and use the resultant income on warm glow. Hiding the dollar lowers tax obligations (and thus raises disposable income) by τ. However, a fraction θ of the dollar is lost, so the net gain to disposable income is $\tau - \theta$. Now consider the effect of hiding a dollar of income on the total amount of the public good. The hidden dollar lowers tax revenue by τ. If the individual spends the increase in disposable income on warm glow, the individual's public-good contribution increases by $\tau - \theta$. Thus, the total amount of the public good falls by θ. By hiding income, individuals can thus increase warm glow but they "pay" for

$w_{inf} = w_f(1-\tau) + \alpha$, the right-hand-side of the constraint would be $(T - s)w_f(1-\tau) + sw_{inf}$. Letting $\alpha = \tau - \theta$, plugging in for w_{inf}, and rearranging gives the same constraint as above where $w_i = Tw_f$ (note the constraint above can be rewritten $w_i(1-\tau) + s_i(\tau - \theta)$).

this increase by seeing a decline in the total amount of the public good. In the standard model, voluntary donations that increase warm glow will also increase public good provision (all else equal). The ability to hide income allows individuals to break this link; individuals can now make warm-glow based decisions that are socially costly.

Now consider an exogenous intervention that increases Y. This will cause the numerator in (3), U_Y^i, to decrease, all else equal. By hiding additional income, individuals can "convert" Y into g_i, thus restoring equality in (3). But since every dollar of Y converted to g_i lowers Y by θ, this conversion process undermines the efficacy of the exogenous intervention. Equation (3) thus identifies a channel through which warm glow incentives can actually lead to crowd out. The following section provides a more rigorous exploration of this intuition.

3. Implications of Hidden Income

3A. Crowd out and Hidden Income

Consider a Nash Equilibrium where individuals are at an interior solution where some income is hidden: $0 < s_i^* < w_i$, where s_i^* is the amount of hidden income solving (1). Interiority is a standard assumption when analyzing the warm glow model; this assumption will be discussed more below.

The above first-order conditions depict an optimal choice of the public good (taking other contributions as given), $Y^* = f^i(w_i, Y_{-i}, \tau, \theta)$. Let $f^i_{Y_{-i}} = \frac{\partial f^i}{\partial Y_{-i}}$. Suppose there is an agent in the economy, agent $n+1$, who voluntarily donates nothing and hides nothing, and consider an increase in the public good financed by increasing taxes on this individual. The equilibrium effect of such a tax increase will be:

$$\frac{dY}{d\tau_{n+1}} = \left(1 + \sum_{i=1}^{n}\left(\frac{1}{f^i_{Y_{-i}}} - 1\right)\right)^{-1} \tag{4}$$

(cf. Andreoni, 1989), which is monotonically increasing in $f^i_{Y_{-i}}$. Larger crowd out corresponds to a smaller value in (4). This equation, and the analysis below, could also considered in the context where an increase in public good provision comes from an exogenous increase in NGO activity. In that case, $Y_{-i} = g_{NGO} + \sum_{k \neq i}(w_k - s_k)\tau + g_k$ for all individuals, where g_{NGO} represents the NGO's provision, and (as shown in the appendix) $\frac{dY}{d\tau_{n+1}} = \frac{dY}{dg_{NGO}}$.

To consider how equation (4) varies with the strength of warm glow, one first needs a measure of the strength of warm glow. The first measure of warm glow considered here is simply an individual's marginal propensity to spend on warm glow in response to a public-good shock, $\frac{\partial g_i^*}{\partial Y_{-i}}$, where g_i^* represents the best-

response voluntary donation which solves (1).[9] Intuitively, if $\frac{\partial g_i^*}{\partial Y_{-i}}$ increases, then individuals will increase their own donations by more (or decrease their donations by a smaller amount) in response to an increase in others' provision of the public good and (in the standard model) this will lead to smaller crowd out.[10]

The model with hidden income is different, however. Before discussing crowd out, one complication with hidden income is choice of private consumption c_i. In the standard model, individuals split their income between donations and private consumption and so a larger propensity to spend income on g_i directly implies a smaller propensity to spend on c_i. With hidden income, this is no longer the case. For example, an individual who responds to an increase in Y_{-i} by increasing the amount of income hidden would be able to have a greater propensity to spend on *both* g_i and c_i than would someone who does not hide additional income after an increase in Y_{-i}. To keep the focus on warm glow incentives, the following proposition holds the propensity to spend income on c_i constant while the letting the propensity to spend on warm glow change. An example of preferences where the propensity to spend on g_i varies but the propensity to spend on c_i does not is given below. Also, a result which relaxes the assumption that $\partial c_i^*/\partial Y_{-i}$ is constant will be considered momentarily. The proposition presents the counter-intuitive result that the *greater* the propensity to spend on warm glow, the *greater* is crowd out:

Proposition 1: *Given τ, w_i, θ, and $\frac{\partial c_i^*}{\partial Y_{-i}}$, greater propensity to spend on warm glow leads to greater crowd out; that is, larger values of $\frac{\partial g_i^*}{\partial Y_{-i}}$ lead to smaller values of $\frac{dY}{d\tau_{n+1}}$.*

Proofs are given in the appendix. Intuitively, if there is an exogenous increase in Y_{-i}, individuals with strong warm glows will exploit the increase in the public good by hiding additional income, essentially transforming this increase in resources into an increase in warm glow. But such a transformation is socially wasteful and leads to crowd out.

The result above considers a readily intuitive concept of warm glow, but also assumes that the propensity to consume the private good c_i does not change as warm glow changes. The following result relaxes this assumption by considering a more general concept of warm glow. Intuitively, the strength of warm glow involves individuals' willingness to substitute private income with external donations to the public good; this second measure of warm glow is based on this notion. Following Andreoni (1989), imagine reducing an individual's endowed income by a dollar and using this dollar to increase the public good. The impact of such a transfer on the individual's preferred amount of public good, all else equal, is represented by $\frac{\partial f^i}{\partial Y_{-i}} - \frac{\partial f^i}{\partial w_i} = f^i_{Y_{-i}} - f^i_{w_i}$; where f^i is an individual's best-response choice of the level of the public good.

[9] Similar results hold if one uses instead the marginal propensity to spend income on warm glow.

[10] This is straightforward to show: in the standard model $f^i = g + \tau w_i + Y_{-i}$ and $f^i_{Y_{-i}} = \partial g_i^*/\partial Y_{-i} + 1$; thus by equation 4 increases in $\partial g_i^*/\partial Y_{-i}$ lead to smaller crowd out.

The measure of warm-glow used next will be this difference, $f^i_{Y_{-i}} - f^i_{w_i}$.

The difference $f^i_{Y_{-i}} - f^i_{w_i}$ corresponds to the term f_{ie}, the "egoistic" derivative of f^i that comes from warm glow, in Andreoni's (1989) analysis; it is straightforward to show that increases in this "egoistic" term lead to smaller crowd out in the standard model.[11] This measure is linked to the earlier measure; if $\frac{\partial g_i^*}{\partial Y_{-i}}$ increases holding $\frac{\partial c_i^*}{\partial Y_{-i}}$ constant, then $f^i_{Y_{-i}} - f^i_{w_i}$ will increase (this is shown in the appendix at the end of the proof for Proposition 2). Initially, it seems like a positive relationship between $\frac{dY}{d\tau_{n+1}}$ and $f^i_{Y_{-i}} - f^i_{w_i}$ should hold "mechanically," since $\frac{dY}{d\tau_{n+1}}$ is monotonically increasing in $f^i_{Y_{-i}}$ (as equation (4) shows) and presumably growth in $f^i_{Y_{-i}}$ would also increase $f^i_{Y_{-i}} - f^i_{w_i}$. However, a negative relationship holds when income is hidden:

Proposition 2: *Given τ, w_i, and θ, larger values of $f^i_{Y_{-i}} - f^i_{w_i}$ lead to smaller values of $\frac{dY}{d\tau_{n+1}}$.*

Proposition 2 shows that the result holds even without making explicit assumptions on the propensity to consume the private good.

These results, as is typically the case in warm-glow analysis, assume an interior solution, including the assumption that individuals choose to hide a fraction of income. Of course, this may not always hold, and some individuals (e.g., those who hide no income at all) may behave as in the standard warm glow model. Suppose there are two types of individuals; type 1 individuals and type 2 individuals. Let f^j be the best-response choice of Y for all type j individuals. Suppose that type 1 individuals' behavior fits the standard warm glow model; for instance, these individuals may not hide any income for ethical reasons or because their income is independently reported. Suppose that type 2s hide some income and fit the assumptions for the propositions here.[12] Letting the fraction of type 1 individuals be ϕ, equation (4) can be written $dY/d\tau_{n+1} = (1 - n + n(\phi/f^1_{Y_{-i}} + (1-\phi)/f^2_{Y_{-i}}))^{-1}$. In the standard model, greater warm glow leads to less crowd out, i.e. larger values of $f^1_{Y_{-i}}$.[13] But for type 2 individuals (as the proofs in the appendix show) stronger warm glow leads to a decrease in $f^2_{Y_{-i}}$. Thus if warm glow increased for both types, the overall effect of stronger warm glow would be a weighted average of the situations using the standard intuition and situation where some income is hidden. As ϕ approaches one the result approaches the standard case and

[11] Intuitively, in the standard no-hidden-income model with no warm glow, individuals view their own income as perfectly substitutable with Y_{-i}. Thus, if private income falls by a dollar and Y_{-i} increases by a dollar, individuals would simply lower g_i by a dollar so that the income transfer would have no effect on the public good: $f^i_{Y_{-i}} - f^i_{w_i} = 0$. With warm glow, individuals view Y_{-i} as an imperfect substitute for private income; thus when a dollar of income is transferred to Y_{-i} private donations g_i fall by less than a dollar, because individuals wish to preserve warm glow. Warm glow incentives thus lead the difference $f^i_{Y_{-i}} - f^i_{w_i}$ to increase. See pages 1451-1453 of Andreoni (1989) for additional discussion of this term in the standard model.

[12] This discussion will take type as fixed, ie, individuals are assumed not to respond to a shock by moving off of (or onto) a corner solution; this could be reasonable for relatively small shocks (or, as noted above, if an individuals' type is determined by factors such as whether wages are independently reported). The appendix shows that an individual who always hides all income (e.g., because of employment in an illicit industry) solves the same optimization problem as a type 1 hide-nothing individual with different endowments of income and Y_{-i}.

[13] As mentioned earlier, the relationship between crowd out and $f^1_{Y_{-i}}$ can be seen in equation (4). The relationship between warm glow and $f^1_{Y_{-i}}$ in the standard model has been discussed in earlier work, cf. Hungerman (2007).

as ϕ approaches zero the result approaches the hidden-income case.

One feature of the model that may influence interiority is the total number of individuals in the community of donors. In particular, as the total number of donors grows, the assumption of an interior solution will become less tenable. Intuitively, if individuals each contribute to the public good in order to get some warm glow for themselves, then the public good may grow without bound as the population increases.[14] This will lead individuals towards corner solutions, as indicated by the representation of the first order conditions in (3). Fortunately, there are some interesting situations—such as situations involving crowd out with interfamily transfers as considered in section 4—where the relevant population of donors may be reasonably small.[15] Before turning to real-world applications, however, the following subsections illustrate the main result using a simple example and consider some additional comparative statics.

3B. An Example

Consider a simple example where individuals' preferences can be represented with a Cobb-Douglas utility function:

$$U^i\left(c_i,\ Y,\ g_i\right) = \alpha_1 \log(c_i) + \alpha_2 \log(Y) + \alpha_3 \log(g_i).$$

Here, larger values of the parameter α_3 represent greater utility derived from warm glow. The agent's best-response choice of the public good in this case is $Y^* = f^i(w_i, Y_{-i}, \tau, \theta) = \frac{\alpha_2(Y_{-i}+w_i\mu)}{(\alpha_1+\alpha_2+\alpha_3)}$, where $\mu = \frac{\tau(1-\theta)}{\tau-\theta}$.[16] This yields the simple expression:

$$\frac{\partial Y^*}{\partial Y_{-i}} = f^i_{Y_{-i}} = \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3} \tag{5}$$

which is decreasing as α_3 grows. Indeed, as the warm glow term α_3 grows increasingly larger the expression in (5) falls towards *zero*, leading to a full-crowd-out result resembling the type of outcome usually ascribed to the standard model when there is no warm glow at all.[17]

Figure 1 illustrates the efficacy of policy intervention in the case of hidden income and in the standard case with no hidden income. Panel A of Figure 1 depicts an equilibrium where individuals can hide income. Here, $n = 5$, $\alpha_1 = 1 - \alpha_2 - \alpha_3$, and $\alpha_2 = 1/2 - \alpha_3$ (along with α_3, these are the only parameters that

[14] A formal discussion of this possibility is given in Ribar and Wilhelm (2002).

[15] Further, there is empirical evidence that under-reporting of income—income is reported but not fully, fitting the interior solution here—is a widespread economic activity, cf. Alm and Yunus (2009), Schneider and Enste (2000), Alm, Bahl, and Murray (1991 and 1990), Poterba (1987), and Clotfelter (1983).

[16] This result it notable for its analytical simplicity, as most warm-glow examples have exceedingly complex solutions. The simplicity is gained from the new first-order condition in equation (3), which involves only two marginal utility terms (instead of the three marginal utility terms in (2)).

[17] In fact, in a pure altruism model the derivative in (5) will actually be greater than the zero effect suggested above, as an increase in Y_{-i} is viewed as an increase in income which (under normality) raises demand for Y.

determine crowd out; these parameter choices are discussed more momentarily).[18] The y-axis shows the equilibrium change in the public good from an exogenous intervention, $\frac{dY}{d\tau_{n+1}}$; i.e., the figure shows the expression in (4) using the result in (5). Larger values on the y-axis correspond to smaller crowd out and more effective policy intervention. The x-axis shows the strength of the warm glow parameter α_3, which varies between 0.1 and 0.3. Panel B of the figure shows equilibrium crowd out when individuals are unable to hide any income; the scale of the y-axes in the two panels is different. In this case crowd out depends upon endowed income and the tax rate; in Panel B $\tau = 0.5$, and $w = 1$.

The figure shows starkly different implications for policy intervention depending upon whether income can be hidden. In the hidden income case, $\frac{dY}{d\tau_{n+1}}$ is diminishing as α_3 increases; this corresponds to increasing crowd out as warm glow incentives grow. Panel B shows that the opposite is true when income cannot be hidden; here crowd out falls (and $\frac{dY}{d\tau_{n+1}}$ rises) as α_3 grows. The figure also highlights a striking difference in the magnitude of crowd out in the two panels. In panel A crowd out is large, while in panel B crowd out is negligible.[19]

3C. Changes in θ and τ

A policy maker concerned with hidden income could attempt to encourage income reporting by decreasing the tax rate or increasing the cost of hiding income. One might wonder how total provision of the public good would change in response to a change in τ or θ. It is in general unclear which parameter would lead to larger changes in the public good's provision. To see this, consider a change in the cost of hiding income, θ, on the equilibrium provision of the public good. Let $y^i = f^i(\bullet) - Y_{-i}$ represent i's total public-good contribution, including both the voluntary and tax components. Totally differentiating this, we have $dy^i = f^i_\theta d\theta + f^i_{Y_{-i}} dY_{-i} - dY_{-i}$. Since $Y_{-i} = Y - y^i$, it follows that $dY_{-i} = dY - dy^i$. Plugging this in for dY_{-i} and collecting terms yields $dy^i = dY\left[(f^i_{Y_{-i}} - 1)/f^i_{Y_{-i}}\right] + \left[f^i_\theta / f^i_{Y_{-i}}\right] d\theta$. Finally, it must be that $dY = \sum_i dy^i$; plugging in for dy^i in this expression yields

$$\frac{dY}{d\theta} = \frac{\sum_i \frac{f^i_\theta}{f^i_{Y_{-i}}}}{1 - \sum_i \left(\frac{f^i_{Y_{-i}} - 1}{f^i_{Y_{-i}}}\right)}. \tag{6}$$

[18] Here larger values of α_3 translate into larger values of $f^i_{Y_{-i}} - f^i_{w_i}$ and $\partial g^*_i / \partial Y_{-i}$, holding $\partial c^*_i / \partial w_i$ constant. Simply setting α_1 and α_2 equal to constants (i.e., not functions of α_3) and letting α_3 increase produces similar results, and would satisfy the conditions of Proposition 2, but in this case $\partial c^*_i / \partial w_i$ may change as α_3 grows.

[19] It can be verified that interior solutions exist for these parameters. Further, while simple, the example produces levels of hidden income and donations broadly consistent with prior work. The solutions in Panel A typically suggest that about half of income is hidden, an amount consistent with the amount of shadow activity reported for many countries in Schneider and Enste (2000); and the fraction of income spent on warm glow is typically around 0.15. This is an amount consistent with some prior findings on the average amount of income transferred by households in developing communities to needy friends and family; see Cox and Fafchamps (2007).

The expression for $\frac{dY}{d\tau}$ is the same as the above except that the numerator contains f_τ^i instead of f_θ^i.

Consider a symmetric equilibrium where $f^i = f$. Using (6), we can compare the effect on the public good of a decrease in the tax rate to the effect of an increase in the cost of hiding resources:

$$\frac{dY}{d\theta} - \left(-\frac{dY}{d\tau}\right) = \frac{dY}{d\theta} + \frac{dY}{d\tau} = \varsigma\left(f_\theta + f_\tau\right) \tag{7}$$

where $\varsigma = 1/\left(1 + (f_{Y_{-i}}/n) - f_{Y_{-i}}\right)$. The term ς will be positive if goods are normal and there is no "crowd in," $0 < f_{Y_{-i}} < 1$. The sign of $\frac{dY}{d\theta} + \frac{dY}{d\tau}$ thus depends on the sign of $f_\theta + f_\tau$.

To explore these partial derivatives of the best response function f, consider the following hypothetical thought experiment. Imagine that the optimization problem faced by individuals in this model was *not* a public-good style problem with two choice variables, but instead was a standard consumer problem with three choice variables, $\max_{c_i, Y, g_i} U^i\left(c_i,\ Y,\ g_i\right) + \lambda(M_i - p_c c_i - p_Y Y - p_g g_i)$, where p_c, p_g, and p_Y were the prices for each good and individuals had income level M_i. This hypothetical problem thus keeps preferences the same but replaces the model's public-good and hidden-income technologies with a simple budget constraint, creating a standard consumer maximization problem. Call the optimal choice of Y in this model $\tilde{Y}^*(M_i, p_c, p_Y, p_g)$; the derivatives of this function $\tilde{Y}^*$ with respect to M_i and prices p_c, p_g, and p_Y can be interpreted as standard income and price effects.

As discussed in the appendix, one can identify values of prices and income (M_i, p_c, p_Y, p_g) such that the solution $\tilde{Y}^*(M_i, p_c, p_Y, p_g)$ will equal the best-response-function choice of the public good $f(w_i, Y_{-i}, \tau, \theta)$. In particular, $\tilde{Y}^*(M_i, p_c, p_Y, p_g)$ will equal $f(w_i, Y_{-i}, \tau, \theta)$ when $p_g = \theta$, $p_Y = \tau - \theta$, $p_c = \tau$, and $M = Y_{-i}(\tau - \theta) + w(1 - \theta)\tau$. These parameter values are intuitive in that at these values the price of good j, p_j, represents the opportunity cost, as measured by foregone public-good provision, from consuming good j; this will be discussed more momentarily. It follows that $\tilde{Y}^*$ can be used to decompose the partial derivatives f_θ and f_τ into traditional price and income effects:

$$f_\theta = -\frac{\partial \tilde{Y}^*}{\partial M}\left(Y_{-i} + w\tau\right) + \frac{\partial \tilde{Y}^*}{\partial p_g} - \frac{\partial \tilde{Y}^*}{\partial p_Y} \tag{8}$$

$$f_\tau = \frac{\partial \tilde{Y}^*}{\partial M}\left(Y_{-i} + w(1-\theta)\right) + \frac{\partial \tilde{Y}^*}{\partial p_c} + \frac{\partial \tilde{Y}^*}{\partial p_Y}. \tag{9}$$

Suppose income effects are small, so that $\frac{\partial \tilde{Y}^*}{\partial M}$ is close to zero. Then the difference for public good provision between lowering the tax rate and raising the cost of hiding income is determined by the simple expression:

$$f_\theta + f_\tau \cong \frac{\partial \tilde{Y}^*}{\partial p_g} + \frac{\partial \tilde{Y}^*}{\partial p_c} \tag{10}$$

which could be either positive or negative and depends upon the cross-price sensitivity of demand for the public good. As noted above, $p_c = \tau$ when $\tilde{Y}^*(M_i, p_c, p_Y, p_g) = f(w_i, Y_{-i}, \tau, \theta)$. Intuitively, an individual who hides a dollar of income for private consumption lowers the public good by τ; the term τ is thus the public-good price of substituting Y for c_i. Decreasing τ will therefore have a large effect on the public good if individuals are willing to substitute between the private good c and the public good; that is, if $\partial\tilde{Y}^* / \partial p_c$ is large.

Alternately, an individual hiding a dollar of income for warm glow lowers the public good by θ, so that θ is the public-good price of substituting Y for g_i. The impact on the public good from increasing θ will thus be relatively large (compared to changing τ) when individuals view warm glow and the public good as especially close substitutes; that is, if $\partial\tilde{Y}^* / \partial p_g$ is large. Thus either changes in θ or changes in τ may have particularly large effects on the provision of the public good, and the effect of changing either of these terms depends upon the extent to which individuals view the public good as substitutable for other types of consumption g_i and c_i.

3D. Tax Revenue

It has been widely acknowledged that tax revenue collection is a critical but challenging issue in developing societies (Besley and Persson, 2013); one might wonder if the results shown earlier have implications for tax revenue as well. This subsection considers how an exogenous increase in the public good would affect tax revenue. Let R denote tax revenue:

$$R = \sum_i (w_i - s_i)\tau.$$

The discussion here will show that an exogenous increase in the public good lowers tax revenue, and this effect is greater as warm glow grows. For purposes of motivation, suppose in this case that the increase in public-good provision was an exogenous increase from an NGO. Then we could express the equilibrium change in tax revenue from this intervention as

$$\frac{dR}{dg_{NGO}} = -\tau \sum_i \frac{\partial s_i}{\partial Y_{-i}} \frac{dY_{-i}}{dg_{NGO}}.$$

We can write $Y_{-i} = g_{NGO} + \sum_{j \neq i} f^j - Y_{-j}$. Totally differentiating this gives: $dY_{-i} = dg_{NGO} + \sum_{j \neq i} \left(f^j_{Y_{-i}} - 1 \right) dY_{-j}$. Suppose the equilibrium were symmetric, so that $f^j = f$ and $dY_{-i} = dY_{-j}$ for all i and j and that s_i is the

same for all i. Then we can write $dY_{-i} = dg_{NGO} + (n-1)\left(f_{Y_{-i}} - 1\right) dY_{-i}$, or

$$\frac{dY_{-i}}{dg_{NGO}} = \left(1 - (n-1)\left(f_{Y_{-i}} - 1\right)\right)^{-1}.$$

Plugging this in, the expression for tax revenue becomes

$$\frac{dR}{dg_{NGO}} = -\tau n \frac{\partial s_i}{\partial Y_{-i}} \left(1 - (n-1)\left(f_{Y_{-i}} - 1\right)\right)^{-1}. \tag{11}$$

The following proposition shows the effect of such a contribution on tax revenue. In addition to symmetry, the proposition takes the same conditions as those in Proposition 1, but adds two further (but mild) assumptions. First, it is assumed there is no "crowd in," so that $0 < f_{Y_{-i}} < 1$. Second, the result adds the mild assumption that $0 \leq \frac{\partial c_i^*}{\partial Y_{-i}}$. This condition is sufficient for proving part (b) of the proposition, but part (a) of the proposition would hold even if this condition were not met. The condition simply says that, given an exogenous increase in i's consumption of the public good, i's best-response choice of private consumption does not go down.

Proposition 3: *Given a symmetric equilibrium and given τ, w_i, θ, and $0 \leq \frac{\partial c_i^*}{\partial Y_{-i}}$, (a) an exogenous increase in the public good will lower tax revenue; that is, $\frac{dR}{dg_{NGO}} < 0$. (b) The greater is warm glow, the greater is the decline in tax revenue; that is, larger values of $\frac{\partial g_i^*}{\partial Y_{-i}}$ correspond to smaller (i.e. more negative) values of $\frac{dR}{dg_{NGO}}$.*

The notion that NGO activity could injuriously affect governance has been raised before (e.g., Bräutigam and Knack, 2004) although the channel here is different. The intuition follows the earlier results: given an exogenous increase in the public good, individuals respond by hiding income, which lowers tax revenue. The greater is the warm glow, the greater is the hidden-income response, and thus the greater the decline in tax revenue.

4. Application to Crowd Out Estimates

The analysis in the prior sections shows that, if individuals choose to hide a fraction of income, then stronger warm glow will lead to more crowd out. Subsection 2 discussed evidence motivating the basic setup of the model, but one may wonder whether this key result has any bearing on empirical estimates of crowd out. This section provides a brief, suggestive exploration of this possibility.

Such an exploration requires three things: (1) estimates of crowd out, (2) a measure of individuals' proclivity to hide income, and (3) a measure of warm glow. Meeting these requirements is potentially challenging; many papers devote considerable resources simply towards producing a measure of crowd out, and I am unaware of any prior work providing estimates of warm glow usable here.

For crowd out estimates, I assembled 22 estimates from 16 different papers in the development literature on crowd out and inter-family transfers. This literature considers whether a change in a household's income (e.g., from a government social-security pension) crowds out informal transfers of income into the household (e.g., remittances sent by adult children).[20] As noted in Warr (1982), transfers of this kind may be considered contributions to a public good.[21] Another advantage of focusing on this literature is that it provides a relatively large number of crowd-out estimates that, arguably, are *prima facie* comparable. An additional benefit is that many papers in this area focus on crowd out in a particular nation, allowing for an international comparison of crowd out in different countries.[22] The estimates show, on a dollar-for-dollar basis, the extent to which a one-dollar increase in a household's income (often from an increase in pension income) reduces private income transfers into the household. Some papers provided a single estimate for the entire sample of available individuals; in that case this "entire sample" estimate would be chosen. Some papers instead provided estimates for different groups separately (e.g., urban/rural); in this case estimates for all subgroups are included.[23]

The second item needed is an estimate of the willingness to hide income. For this I turn to the World Values Survey (WVS), a survey conducted roughly every five years for about 90 countries (not every country is surveyed every year). The survey has consisted of five waves: 1981-84 (wave 1), 1989-93 (wave 2), 1994-99 (wave 3), 1999-2004 (wave 4), and 2005-07 (wave 5). The survey includes a question where individuals rate on a scale of 1 to 10 how "justifiable" it is to cheat on taxes (1 = "never justifiable", 10 = "always justifiable"); this question has been asked in every wave of the WVS. I simply take the fraction of individuals in a country reporting that it is ever justifiable to cheat on taxes (ie, those giving an answer of 2 or greater).[24]

The final item required is a measure of warm glow. Remarkably, the WVS contains two questions which

[20] One might be concerned that the set of papers here was chosen (even inadvertently) in a way to fit the model. A subset of these papers are listed in a handbook chapter by Cox and Fafchamps (2007); restricting the analysis to this "exogenous" list of papers reduces the sample but produces results qualitatively consistent with those shown here.

[21] Moreover, as discussed in (e.g.) Cox and Jimenez (1992), Cox, Eser, and Jimenez (1998), Albarran and Attanasio (2002), and Cox and Fafchamps (2007), most transfers of this nature take place between close relatives; thus, for a decision to give to a family member the relevant population of donors would often be other close relatives, a group small enough that the asymptotic concerns raised in the prior section would plausibly not apply. Several papers provide information on the size of households among both donors and recipients of transfers (e.g., Cox, Hansen, Jimenez, 2004; Lai and Orsuwan, 2009; Fan, 2010; Juarez, 2009); the typical size is often comparable to the population size in the example used earlier.

[22] The papers here exclude studies in countries not included in the World Values Survey (e.g., Botswana and Fiji).

[23] In most cases the "baseline" crowd out estimate was readily identifiable and was used, but a few cases report multiple estimates. In general, the results are robust to trading a "robustness" estimate for a "baseline" estimate in a particular paper.

[24] An alternative method of measuring hidden income would be to take measures of the size of a nation's underground economy as in Schneider and Buehn (2009). For some countries here, I am unaware of estimates of the shadow economy; the WVS measure above thus allows a greater sample size. However, using shadow-economy estimates for the subset of countries where they are available produces results close to those shown here.

are closely-related to the idea of warm glow. On a 1 to 4 scale (with 1 being strongly disagree and 4 being strongly agree), individuals evaluate the statement: "I would give part of my income if I were certain that the money would be used to prevent environmental pollution." Individuals then use this scale to evaluate the statement: "I would agree to an increase in taxes if the extra money were used to prevent environmental pollution."[25] In the warm-glow model the distinction between willingness to donate taxes and willingness to donate income is captured by the strength of warm-glow preferences: individuals willing to donate income but unwilling to donate taxes are individuals with high warm glow. Accordingly, the measure of warm glow here is simply the answer to the first question minus the answer to the second, resulting in a variable that ranges from 3 to -3. Higher values correspond to a stronger warm glow.[26]

One concern about this measure of warm glow is that differences in the willingness to pay taxes and to donate income may stem from individuals' perception of government efficiency. The WVS also asks individuals to evaluate, on a 1-5 scale, their confidence in the government. I regress each individual's warm glow measure on (a) a set of dummies for country of residence (b) a set of 5 dummies for confidence in the government, and (c) a set of dummies for survey year. I use the coefficients from the country dummies as the measure of warm glow. These coefficients show, controlling for confidence in the government, the average level of warm glow in a country. Simply using the average level of warm glow in a country (not controlling for confidence in government) produces results qualitatively similar to those shown here.[27]

Table 1 shows the list of papers, the nations in each study, the fraction saying cheating is justifiable in each country, average warm glow (relative to Poland, the omitted country) and estimated crowd out of inter-family transfers. The table is arranged from the country with the fewest portion of individuals saying cheating is justifiable (Bangladesh, by far) to the country with the most saying it is justifiable (the Philippines).[28] There is notable variation across all the variables. The warm glow estimates range by about 0.3, which is little less than half a standard deviation. The crowd-out estimates also display some variety; most are negative and between zero and -0.3 (with clear exceptions).

[25] In fact, the original scales in the raw data were flipped, with 1 as strongly agree and 4 as strongly disagree. The scales are reversed here simply to make the measure of warm glow more intuitive.

[26] This question was not asked in the 1981-1984 wave and the 1994-1999 wave of the survey. Redoing the measure of cheating on taxes only for the waves where the warm glow measure is available produces similar results.

[27] One might note that this warm glow measure uses questions on pollution whereas the crowd-out estimates look at inter-family transfers. To the extent that the measure of warm glow used here is specific to pollution, this would presumably lead to spurious noise in the analysis and work against identifying any relationship between warm glow and family transfers. However, there is at least some reason to believe that preferences stated here are informative for non-pollution-related charitable activities. The World Values Survey asks individuals if they are members of organizations which assist the handicapped, religious organizations, human rights organizations, and other community and voluntary groups. One can thus construct a dummy variable that equals unity if an individual is a member of a local voluntary or community organization and regress this dummy on the two questions used to construct the warm glow measure. These regressions show that greater willingness to pay income and greater willingness to pay taxes to fight pollution are both statistically significantly associated with greater levels of voluntarism in organizations. The coefficient on the donate income question is 0.022 [robust se = 0.005], so that a one-digit increase in an individual's willingness to donate income to fight pollution raises the likelihood of any community voluntarism by 2 percentage points. The coefficient on the willingness to pay taxes is 0.011 [0.005].

[28] Andreoni, Erard, and Feinstein (1998) note high tax evasion rates in the Philippines, consistent with the table here.

Looking at Table 1, there is a break in the fraction saying cheating is justifiable between China and Peru. Taking Peru, Taiwan, South Africa, Burkina Faso, Poland, Mexico and the Philippines as the "hiding income" group, the model here suggests that the relationship between warm glow and crowd out should be attenuated or even negative for this group. (The result here is not overly sensitive to this choice of cutoff.) Meanwhile, if countries with low levels of hidden income are better described by the standard model, the relationship between crowd out and warm glow for these countries should be positive.[29]

Figure 2 plots the relationship between crowd out and warm glow for the "more hiding" and "less hiding" groups. Consistent with the standard model, countries with less hiding have a positive relationship between average warm glow and published crowd-out estimates.[30] In contrast, the "more hiding" group shows a *negative* relationship: countries where individuals have stronger warm glow are countries were published crowd out estimates tend to be further below zero.[31]

These findings in Figure 2 should be regarded as suggestive, as they concern a small sample of countries and the magnitude of the observed effect would be sensitive to the inclusion of a few larger "outlier" oubservations. Also, the data here attempt to control for mitigating issues such as government corruption in a very simple (albeit transparent) way. However, the results nonetheless indicate that the relationship between crowd out and warm glow may be very different in different settings. Moreover, in situations where hidden income is a common occurrence the simple analysis here suggests that, in accordance with the model, greater warm glow may in fact be associated with greater crowd out.

5. Conclusions

This paper presents a warm glow analysis in a setting where individuals can hide income from a government or other central authority. Prior work has suggested that hiding income is a widespread phenomenon, including in situations where public goods are jointly provided. The findings of this paper suggest that the effects of warm glow and the efficacy of policy intervention may be quite different in these situations: when individuals can make costly decisions to hide income, warm glow incentives can amplify one's willingness to undertake actions that undermine policy interventions. This paper thus highlights a need for continued

[29]The earlier analysis contrasted a case where all individuals hide some income to the case where no one hides while the comparison here is between places with more hiding and less; but, as noted in section 3, the earlier analysis extends to a more-hiding-versus-less-hiding comparison also.

[30]The less-hiding group includes a coefficient of almost positive one, but removing this estimate does not change the positive relationship. Indeed, the trends in this picture are not driven by any particular study.

[31]One might wonder whether the relationship between warm glow and crowd out is statistically significantly different for the two groups. Using a simple OLS regression of crowd out on (a) a constant (b) warm glow (c) a dummy for being in the more-hiding group, and (d) the interaction of (b) and (c), one can reject the hypothesis that the slopes of the two lines are equal (the p-value from this test is 0.017 and is based on t-statistics constructed using robust standard errors). This result is robust to dropping any one paper from the sample, or even (e.g.) dropping the Raut and Tran outlier and the two papers based on Mexico. The results are also preserved if one collapses the data to country-level averages.

work on the joint provision of public goods in settings where income concealment may be common, as in developing economies.

One simplification in the present paper is that the cost of evading taxation is wholly captured in the term θ. As discussed in Andreoni, Erard, and Feinstein (1998) and Slemrod (2007), individuals may face both intrinsic incentives that affect the decision to hide income (such as guilt, shame, and civic virtue) and explicit incentives (such as the expected monetary cost of hiding income or the probability of audit); these incentives could interact with each other in complex ways (see also Frey, 1997). An analysis of how different types of incentives for hiding income interact when determining public-good provision is left for future work.

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Appendix

Proof of Proposition 1

Rewrite the budget constraint $g_i = w_i(1-\tau) + s_i(\tau-\theta) - c_i$ and differentiate:

$$\frac{\partial g_i}{\partial Y_{-i}} = \frac{\partial s_i}{\partial Y_{-i}}(\tau-\theta) - \frac{\partial c_i}{\partial Y_{-i}} \tag{12}$$

By assumption $\frac{\partial c_i}{\partial Y_{-i}}$ is constant, so that an increase in $\frac{\partial g_i}{\partial Y_{-i}}$ corresponds to an increase in $\frac{\partial s_i}{\partial Y_{-i}}$. The public good can be expressed as $Y = Y_{-i} + (w_i - s_i)\tau + g_i$; differentiating this with respect to Y_{-i} and using (12) yields $\frac{\partial Y^*}{\partial Y_{-i}} = f^i_{Y_{-i}} = 1 - \frac{\partial s_i^*}{\partial Y_{-i}}\theta - \frac{\partial c_i^*}{\partial Y_{-i}}$. Since $\frac{\partial c_i}{\partial Y_{-i}}$ is constant and $\frac{\partial s_i}{\partial Y_{-i}}$ has increased, $f^i_{Y_{-i}}$ has decreased. It follows by equation (4) that $\frac{dY}{d\tau_{n+1}}$ has decreased.

Extension to Non-linear Costs

The above result assumes that the marginal cost of hiding income is constant; consider relaxing this assumption. With nonlinearity, individuals who chose to hide different amounts of income (e.g., to consume different levels of c_i) would face different marginal costs of converting Y to g_i; this could lead to differences in crowd out behavior even absent differences in warm glow. To address this complication, consider for simplicity an equilibrium outcome with a given level of hidden income s^*. Suppose that the marginal cost of hiding income was positive and less than τ (but not necessarily constant). Specifically, let the cost of hiding income be given by the differentiable function $h(s)$, $0 < \frac{\partial h}{\partial s} < \tau$. Here, the budget constraint is $g_i = w_i(1-\tau) + s_i\tau - h(s_i) - c_i$. Differentiating yields: $\frac{\partial g_i}{\partial Y_{-i}} = \frac{\partial s_i}{\partial Y_{-i}}\left(\tau - \frac{\partial h}{\partial s_i}\right) - \frac{\partial c_i}{\partial Y_{-i}}$. If $\frac{\partial c_i}{\partial Y_{-i}}$ is constant and $\frac{\partial h}{\partial s_i}$ is less than τ, then as before an increase in $\frac{\partial g_i}{\partial Y_{-i}}$ corresponds to an increase in $\frac{\partial s_i}{\partial Y_{-i}}$. Following the above proof, we have $\frac{\partial Y^*}{\partial Y_{-i}} = f^i_{Y_{-i}} = 1 - \frac{\partial h}{\partial s_i}\frac{\partial s_i^*}{\partial Y_{-i}} - \frac{\partial c_i^*}{\partial Y_{-i}}$. For a given $\frac{\partial c_i}{\partial Y_{-i}}$ and s^*, and since $\frac{\partial h}{\partial s_i}$ is positive, if $\frac{\partial s_i}{\partial Y_{-i}}$ has increased, $f^i_{Y_{-i}}$ again has decreased and thus $\frac{dY}{d\tau_{n+1}}$ has decreased.

Equating $\frac{dY}{d\tau_{n+1}}$ and $\frac{dY}{dg_{NGO}}$

In this case, $Y_{-i} = g_{NGO} + \sum_{k\neq i}(w_k - s_k)\tau + g_k$ for all i. As in section 3C, let $y^i = f^i(\bullet) - Y_{-i}$, and $dy^i = f^i_{Y_{-i}}dY_{-i} - dY_{-i}$. As before, $dY_{-i} = dY - dy^i$, using this for dY_{-i} yields $dy^i = f^i_{Y_{-i}}\left(dY - dy^i\right) - \left(dY - dy^i\right)$ or $dy^i = dY\left(f^i_{Y_{-i}} - 1\right)/f^i_{Y_{-i}}$. Next, $dY = dg_{NGO} + \sum_i dy^i = dg_{NGO} + \sum_i dY\left(f^i_{Y_{-i}} - 1\right)/f^i_{Y_{-i}}$, or $dY\left(1 - \sum_i\left(f^i_{Y_{-i}} - 1\right)/f^i_{Y_{-i}}\right) = dg_{NGO}$. Dividing out and multiplying the negative one through gives $\frac{dY}{dg_{NGO}} = \left(1 + \sum_{i=1}^n\left(\frac{1}{f^i_{Y_{-i}}} - 1\right)\right)^{-1}$, which matches equation (4).

Proof of Proposition 2

The proof begins by establishing that $f^i_{Y_{-i}} = \frac{\tau-\theta}{\tau-\tau\theta}f^i_{w_i}$. As discussed in section 2, the individual's objective function can be written $U^i\left(w_i - s_i\theta + Y_{-i} - Y,\ Y,\ Y - Y_{-i} - (w_i - s_i)\tau\right)$. As noted in the text, one of the first order conditions can be represented as $\theta U^i_Y = (\tau-\theta)U^i_{g_i}$; differentiating this with respect to Y_{-i} yields:

$$\theta U^i_{Yc_i}\left(-\theta\tfrac{\partial s_i}{\partial Y_{-i}}+1-f^i_{Y_{-i}}\right)+\theta U^i_{YY}f^i_{Y_{-i}}+\theta U^i_{Yg_i}\left(f^i_{Y_{-i}}-1+\tau\tfrac{\partial s_i}{\partial Y_{-i}}\right)$$
$$=(\tau-\theta)\,U^i_{g_ic_i}\left(-\theta\tfrac{\partial s_i}{\partial Y_{-i}}+1-f^i_{Y_{-i}}\right)+(\tau-\theta)\,U^i_{g_iY}f^i_{Y_{-i}}+(\tau-\theta)\,U^i_{g_ig_i}\left(f^i_{Y_{-i}}-1+\tau\tfrac{\partial s_i}{\partial Y_{-i}}\right)$$

Gathering terms, this becomes:

$$\begin{aligned}&\tfrac{\partial s_i}{\partial Y_{-i}}\left(-\theta^2U^i_{Yc_i}+\theta\tau U^i_{Yg_i}+\theta\,(\tau-\theta)\,U^i_{g_ic_i}-\tau\,(\tau-\theta)\,U^i_{g_ig_i}\right)\\&\qquad+f^i_{Y_{-i}}\left(-\theta U^i_{Yc_i}+\theta U^i_{YY}+\theta U^i_{Yg_i}+(\tau-\theta)\,U^i_{g_ic_i}-(\tau-\theta)\,U^i_{g_iY}-(\tau-\theta)\,U^i_{g_ig_i}\right)\\&\qquad=(\tau-\theta)\,U^i_{g_ic_i}-(\tau-\theta)\,U^i_{g_ig_i}-\theta U^i_{Yc_i}+\theta U^i_{Yg_i}\end{aligned}$$

Differentiating the first order condition, $\tau U^i_{g_i}=\theta U^i_{c_i}$, with respect to Y_{-i} yields

$$\tau U^i_{g_ic_i}\left(-\theta\tfrac{\partial s_i}{\partial Y_{-i}}+1-f^i_{Y_{-i}}\right)+\tau U^i_{g_iY}f^i_{Y_{-i}}+\tau U^i_{g_ig_i}\left(f^i_{Y_{-i}}-1+\tau\tfrac{\partial s_i}{\partial Y_{-i}}\right)$$
$$=\theta U^i_{c_ic_i}\left(-\theta\tfrac{\partial s_i}{\partial Y_{-i}}+1-f^i_{Y_{-i}}\right)+\theta U^i_{c_iY}f^i_{Y_{-i}}+\theta U^i_{c_ig_i}\left(f^i_{Y_{-i}}-1+\tau\tfrac{\partial s_i}{\partial Y_{-i}}\right)$$

This can be rewritten:

$$\begin{aligned}&\tfrac{\partial s_i}{\partial Y_{-i}}\left(-\theta\tau U^i_{g_ic_i}+\tau^2U^i_{g_ig_i}+\theta^2U^i_{c_ic_i}-\theta\tau U^i_{c_ig_i}\right)\\&\qquad+f^i_{Y_{-i}}\left(-\tau U^i_{g_ic_i}+\tau U^i_{g_iY}+\tau U^i_{g_ig_i}+\theta U^i_{c_ic_i}-\theta U^i_{c_iY}-\theta U^i_{c_ig_i}\right)\\&\qquad=\theta U^i_{c_ic_i}-\theta U^i_{c_ig_i}-\tau U^i_{g_ic_i}+\tau U^i_{g_ig_i}\end{aligned}$$

This suggests a system with two equations and two unknowns:

$$\begin{bmatrix} a_1 & \alpha_1\\ a_2 & \alpha_2\end{bmatrix}\begin{bmatrix}\tfrac{\partial s_i}{\partial Y_{-i}}\\ f^i_{Y_{-i}}\end{bmatrix}=\begin{bmatrix} b_1\\ b_2\end{bmatrix}$$

where

$$\begin{aligned}a_1&=-\theta^2U^i_{Yc_i}+\theta\tau U^i_{Yg_i}+\theta\,(\tau-\theta)\,U^i_{g_ic_i}-\tau\,(\tau-\theta)\,U^i_{g_ig_i}\\ \alpha_1&=-\theta U^i_{Yc_i}+\theta U^i_{YY}+\theta U^i_{Yg_i}+(\tau-\theta)\,U^i_{g_ic_i}-(\tau-\theta)\,U^i_{g_iY}-(\tau-\theta)\,U^i_{g_ig_i}\\ b_1&=(\tau-\theta)\,U^i_{g_ic_i}-(\tau-\theta)\,U^i_{g_ig_i}-\theta U^i_{Yc_i}+\theta U^i_{Yg_i}\\ a_2&=-\theta\tau U^i_{g_ic_i}+\tau^2U^i_{g_ig_i}+\theta^2U^i_{c_ic_i}-\theta\tau U^i_{c_ig_i}\\ \alpha_2&=-\tau U^i_{g_ic_i}+\tau U^i_{g_iY}+\tau U^i_{g_ig_i}+\theta U^i_{c_ic_i}-\theta U^i_{c_iY}-\theta U^i_{c_ig_i}\\ b_2&=\theta U^i_{c_ic_i}-\theta U^i_{c_ig_i}-\tau U^i_{g_ic_i}+\tau U^i_{g_ig_i}\end{aligned}$$

Solving using Cramer's Rule (or basic algebra) yields:

$$f^i_{Y_{-i}}=(a_1b_2-a_2b_1)\,/\,(a_1\alpha_2-a_2\alpha_1)\,.$$

Next, we find $f^i_{w_i}$. Differentiating $\theta U^i_Y = (\tau-\theta)U^i_{g_i}$ with respect to w_i yields:

$$\begin{aligned}\theta U^i_{Yc_i}\left(-\theta\tfrac{\partial s_i}{\partial w_i}+1-f^i_{w_i}\right)+\theta U^i_{YY}f^i_{w_i}+\theta U^i_{Yg_i}\left(f^i_{w_i}-\tau+\tau\tfrac{\partial s_i}{\partial w_i}\right)\\ =(\tau-\theta)\,U^i_{g_ic_i}\left(-\theta\tfrac{\partial s_i}{\partial w_i}+1-f^i_{w_i}\right)+(\tau-\theta)\,U^i_{g_iY}f^i_{w_i}+(\tau-\theta)\,U^i_{g_ig_i}\left(f^i_{w_i}-\tau+\tau\tfrac{\partial s_i}{\partial w_i}\right)\end{aligned}$$

Gathering terms,

$$\begin{aligned}\tfrac{\partial s_i}{\partial w_i}\left(-\theta^2U^i_{Yc_i}+\theta\tau U^i_{Yg_i}+\theta\,(\tau-\theta)\,U^i_{g_ic_i}-\tau\,(\tau-\theta)\,U^i_{g_ig_i}\right)\\ +f^i_{w_i}\left(-\theta U^i_{Yc_i}+\theta U^i_{YY}+\theta U^i_{Yg_i}+(\tau-\theta)\,U^i_{g_ic_i}-(\tau-\theta)\,U^i_{g_iY}-(\tau-\theta)\,U^i_{g_ig_i}\right)\\ =(\tau-\theta)\,U^i_{g_ic_i}-\tau\,(\tau-\theta)\,U^i_{g_ig_i}-\theta U^i_{Yc_i}+\theta\tau U^i_{Yg_i}\end{aligned}$$

Or, $\frac{\partial s_i}{\partial w_i}a_1+f^i_{w_i}\alpha_1=b_3$. Finally, differentiating the first order condition, $\tau U^i_{g_i}=\theta U^i_{c_i}$, with respect to w_i:

$$\begin{aligned}\tau U^i_{g_ic_i}\left(-\theta\tfrac{\partial s_i}{\partial w_i}+1-f^i_{w_i}\right)+\tau U^i_{g_iY}f^i_{w_i}+\tau U^i_{g_ig_i}\left(f^i_{w_i}-\tau+\tau\tfrac{\partial s_i}{\partial w_i}\right)\\ =\theta U^i_{c_ic_i}\left(-\theta\tfrac{\partial s_i}{\partial w_i}+1-f^i_{w_i}\right)+\theta U^i_{c_iY}f^i_{w_i}+\theta U^i_{c_ig_i}\left(f^i_{w_i}-\tau+\tau\tfrac{\partial s_i}{\partial w_i}\right)\end{aligned}$$

This can be rewritten:

$$\begin{aligned}\tfrac{\partial s_i}{\partial w_i}\left(-\theta\tau U^i_{g_ic_i}+\tau^2U^i_{g_ig_i}+\theta^2U^i_{c_ic_i}-\theta\tau U^i_{c_ig_i}\right)\\ +f^i_{w_i}\left(-\tau U^i_{g_ic_i}+\tau U^i_{g_iY}+\tau U^i_{g_ig_i}+\theta U^i_{c_ic_i}-\theta U^i_{c_iY}-\theta U^i_{c_ig_i}\right)\\ =\theta U^i_{c_ic_i}-\theta\tau U^i_{c_ig_i}-\tau U^i_{g_ic_i}+\tau^2U^i_{g_ig_i}\end{aligned}$$

Or, $\frac{\partial s_i}{\partial w_i}a_2+f^i_{w_i}\alpha_2=b_4$. We again have two equations and two unknowns:

$$\begin{bmatrix} a_1 & \alpha_1\\ a_2 & \alpha_2\end{bmatrix}\begin{bmatrix}\frac{\partial s_i}{\partial w_i}\\ f^i_{w_i}\end{bmatrix}=\begin{bmatrix} b_3\\ b_4\end{bmatrix}$$

where

$$\begin{aligned}b_3&=(\tau-\theta)\,U^i_{g_ic_i}-\tau\,(\tau-\theta)\,U^i_{g_ig_i}-\theta U^i_{Yc_i}+\theta\tau U^i_{Yg_i}\\ b_4&=\theta U^i_{c_ic_i}-\theta\tau U^i_{c_ig_i}-\tau U^i_{g_ic_i}+\tau^2U^i_{g_ig_i}\end{aligned}$$

and the other terms are defined as before. The solution is

$$f^i_{w_i}=(a_1b_4-a_2b_3)\,/\,(a_1\alpha_2-a_2\alpha_1)\,.$$

We will show that $f^i_{Y_{-i}}=\left(\frac{\tau-\theta}{\tau-\tau\theta}\right)f^i_{w_i}$, that is, that $a_1b_2-a_2b_1=\left(\frac{\tau-\theta}{\tau-\tau\theta}\right)(a_1b_4-a_2b_3)$ or equivalently that $a_1\,(b_2-\varsigma b_4)=a_2\,(b_1-\varsigma b_3)$, where $\varsigma=\left(\frac{\tau-\theta}{\tau-\tau\theta}\right)$. Focusing on the left-hand side of the equation, we

have

$$b_2 - \varsigma b_4 = \theta U^i_{c_i c_i} - (\theta + \tau) U^i_{c_i g_i} + \tau U^i_{g_i g_i} - \left(\frac{\tau - \theta}{\tau - \tau\theta}\right)\left(\theta U^i_{c_i c_i} - (\tau + \theta\tau) U^i_{c_i g_i} + \tau^2 U^i_{g_i g_i}\right)$$

where we have exploited the fact that $U_{\ell k} = U_{k\ell}$ and alphabetized all cross-partial subscripts. Gathering terms, this can be expressed as

$$\begin{aligned} b_2 - \varsigma b_4 &= \theta U^i_{c_i c_i}\left(\tfrac{\tau-\tau\theta-\tau+\theta}{\tau(1-\theta)}\right) - U^i_{c_i g_i}\left(\tfrac{(\theta+\tau)(\tau-\theta\tau)-(\tau-\theta)(\tau+\theta\tau)}{\tau(1-\theta)}\right) + \tau U^i_{g_i g_i}\left(\tfrac{\tau-\tau\theta-\tau^2+\theta\tau}{\tau(1-\theta)}\right) \\ &= \tfrac{1-\tau}{1-\theta}\left(\tfrac{\theta^2}{\tau} U^i_{c_i c_i} + \tau U^i_{g_i g_i} - 2\theta U^i_{c_i g_i}\right) \end{aligned}$$

And so

$$\begin{aligned} a_1 (b_2 - \varsigma b_4) = &\left(-\theta^2 U^i_{c_i Y} + \theta\tau U^i_{g_i Y} + \theta(\tau - \theta) U^i_{c_i g_i} - \tau(\tau - \theta) U^i_{g_i g_i}\right) \\ &\times \frac{1-\tau}{1-\theta}\left(\frac{\theta^2}{\tau} U^i_{c_i c_i} + \tau U^i_{g_i g_i} - 2\theta U^i_{c_i g_i}\right) \end{aligned} \tag{13}$$

Consider next $a_2 (b_1 - \varsigma b_3)$, starting with the terms $b_1 - \varsigma b_3$:

$$b_1 - \varsigma b_3 = (\tau - \theta) U^i_{c_i g_i} - (\tau - \theta) U^i_{g_i g_i} - \theta U^i_{c_i Y} + \theta U^i_{g_i Y} - \left(\frac{\tau - \theta}{\tau - \tau\theta}\right)\left((\tau - \theta) U^i_{c_i g_i} - \tau(\tau - \theta) U^i_{g_i g_i} - \theta U^i_{c_i Y} + \theta\tau U^i_{g_i Y}\right)$$

where again subscripts have been alphabetized. Collecting terms, this becomes

$$\begin{aligned} b_1 - \varsigma b_3 &= (\tau - \theta) U^i_{c_i g_i}\left(\tfrac{\tau-\theta\tau-\tau+\theta}{\tau(1-\theta)}\right) - (\tau - \theta) U^i_{g_i g_i}\left(\tfrac{\tau-\theta\tau-\tau^2+\theta\tau}{\tau(1-\theta)}\right) - \theta U^i_{c_i Y}\left(\tfrac{\tau-\theta\tau-\tau+\theta}{\tau(1-\theta)}\right) + \theta U^i_{g_i Y}\left(\tfrac{\tau-\theta\tau-\tau^2+\theta\tau}{\tau(1-\theta)}\right) \\ &= \tfrac{1-\tau}{1-\theta}\left(\tfrac{(\tau-\theta)\theta}{\tau} U^i_{c_i g_i} - (\tau - \theta) U^i_{g_i g_i} - \tfrac{\theta^2}{\tau} U^i_{c_i Y} + \theta U^i_{g_i Y}\right) \end{aligned}$$

And so,

$$a_2 (b_1 - \varsigma b_3) = \left(\tau^2 U^i_{g_i g_i} + \theta^2 U^i_{c_i c_i} - 2\theta\tau U^i_{c_i g_i}\right)\frac{1-\tau}{1-\theta}\left(\frac{(\tau - \theta)\theta}{\tau} U^i_{c_i g_i} - (\tau - \theta) U^i_{g_i g_i} - \frac{\theta^2}{\tau} U^i_{c_i Y} + \theta U^i_{g_i Y}\right)$$

Multiplying the rightmost set of parentheses by τ and dividing the leftmost set of parentheses by τ yields

$$a_2 (b_1 - \varsigma b_3) = \left(\tau U^i_{g_i g_i} + \frac{\theta^2}{\tau} U^i_{c_i c_i} - 2\theta U^i_{c_i g_i}\right)\frac{1-\tau}{1-\theta}\left((\tau - \theta)\theta U^i_{c_i g_i} - \tau(\tau - \theta) U^i_{g_i g_i} - \theta^2 U^i_{c_i Y} + \theta\tau U^i_{g_i Y}\right)$$

which matches equation (13). Thus, $a_2 (b_1 - \varsigma b_3) = a_1 (b_2 - \varsigma b_4)$, or $a_1 b_2 - a_2 b_1 = \varsigma (a_1 b_4 - a_2 b_3)$, or

$$f^i_{Y_{-i}} = \frac{a_1 b_2 - a_2 b_1}{a_1\alpha_2 - a_2\alpha_1} = \frac{\varsigma (a_1 b_4 - a_2 b_3)}{a_1\alpha_2 - a_2\alpha_1} = \varsigma f^i_{w_i} \tag{14}$$

where $\varsigma = \left(\frac{\tau-\theta}{\tau-\tau\theta}\right)$. The difference $f^i_{Y_{-i}} - f^i_{w_i}$ can then be expressed $f^i_{Y_{-i}}(1 - 1/\varsigma) = f^i_{Y_{-i}}\left(1 - \left(\frac{\tau-\tau\theta}{\tau-\theta}\right)\right)$.

The expression is negative; an increase in this value corresponds to a decrease in $f^i_{Y_{-i}}$. As $f^i_{Y_{-i}}$ decreases, it follows by equation (4) that $\frac{dY}{d\tau_{n+1}}$ decreases, and the proposition is established.

The intuition for this result stems not from any noteworthy restriction on preferences but instead from the choice set available when income can be hidden. Figure A1 illustrates this choice set, which consists of the trapezoid ABCD. Suppose an individual, taking Y_{-i} as given, chooses to hide all income and spend all resources on c_i. This corner solution is located at point A in the picture. If an individual hiding all income spends everything on warm glow, the individual would be at point C; the line AC represents consumption choices when all income is hidden. The line BD represents consumption choices when no income is hidden; BD is the budget line in the standard model with no hidden income. Points interior to ABCD represent solutions where some income is hidden.

Note that an exogenous increase in Y_{-i} would move the budget plane to the "right", shifting the points A, B, C, and D, rightward along the Y axis. An increase in income w_i would shift the budget plane outward along all three axes at once (i.e., the point B would be further out on both the c and the Y axes than before, and the point D would be further out on both the g and the Y axes than before). For individuals along the edges of the plane, shifts in Y_{-i} and w_i will not be equivalent, but for individuals at *interior* solutions, which are now possible because of hidden income, changes in income will be viewed as equivalent to appropriately chosen increases in the endowment of the public good (or increases in the private good, for that matter). This equivalency drives the proposition (and Proposition 1 as well).[32]

Finally, the proof of Proposition 1 shows that if $\frac{\partial g_i}{\partial Y_{-i}}$ increases, holding $\frac{\partial c_i}{\partial Y_{-i}}$ constant, then $f^i_{Y_{-i}}$ decreases. Then $f^i_{Y_{-i}} - f^i_{w_i} = f^i_{Y_{-i}}\left(1 - \left(\frac{\tau-\tau\theta}{\tau-\theta}\right)\right)$ must increase. An increase in the warm-glow concept used in Proposition 1 thus implies increasing warm glow here.

Individuals who Hide All Income

As mentioned in section 3, in a setting where there are type 1 individuals hiding nothing and type 2 individuals hiding some income, individuals who hide all income would solve the same optimization problem as a certain type 1 hide-nothing individual facing a different endowment of income and Y_{-i}. A "hide everything" individual j with $s_j = w_j$ solves $\max_Y \; U^j\left(w_j(1-\theta) + Y_{-j} - Y,\ Y,\ Y - Y_{-j}\right)$. A hide-nothing individual i solves $\max_Y \; U^i\left(w_i + Y_{-i} - Y,\ Y,\ Y - Y_{-i} - w_i\tau\right)$. These optimization problems are equivalent if $Y_{-i} + w_i\tau = Y_{-j}$ and $w_i + Y_{-i} = w_j(1-\theta) + Y_{-j}$; the latter can be rewritten $w_i = w_j(1-\theta)/(1-\tau)$. Also, letting $\widetilde{Y_{-j}} = Y_{-i} + w_i\tau$ be the choice of Y_{-j} that makes these problems equivalent, and similarly for $\widetilde{w_j}$, it

[32]This suggests an alternate proof. Consider an exogenous increase in Y_{-i} to $\hat{Y}_{-i}$. Denote the level of the public good that now solves the individual's maximization problem $\hat{Y}^* = f^i(w_i, \hat{Y}_{-i}, \tau, \theta)$. One could identify a level of income, $\bar{w}_i$, that, for the original value of Y_{-i}, would yield $\hat{Y}^*$ (and would also yield the associated optimal choices $\hat{c}^*_i$ and $\hat{g}^*_i$). That is, one could identify a $\bar{w}_i$ so that $\bar{Y}^* = f^i(\bar{w}_i, Y_{-i}, \tau, \theta) = \hat{Y}^*$, and where $\bar{c}^*_i = \hat{c}^*_i$ and $\bar{g}^*_i = \hat{g}^*_i$. Imposing these equalities using the expressions for c and g in (1) yields $\bar{w}_i - w_i = \varsigma\left(\hat{Y}_{-i} - Y_{-i}\right)$, which then implies the result in (14). A third derivation of this result is discussed below.

follows that $f^i(w_i, Y_{-i}, \tau, \theta) = f^j(\widetilde{w}_j, \widetilde{Y}_{-j}, \tau)$ and, since $\frac{\partial \widetilde{Y_{-j}}}{\delta Y_{-i}} = 1$, we further have that $f^i_{Y_{-i}}(w_i, Y_{-i}, \tau, \theta) = f^j_{Y_{-j}}(\widetilde{w}_j, \widetilde{Y}_{-j}, \tau)\frac{\partial \widetilde{Y_{-j}}}{\delta Y_{-i}} = f^j_{Y_{-j}}(\widetilde{w}_j, \widetilde{Y}_{-j}, \tau)$.

Deriving f_θ and f_τ

The optimization problem in (1) is depicted as having two choice variables, Y and s_i . However, the utility function has three components: c_i, Y, and g_i. Given a solution to the problem in (1), there exists a corresponding *three variable* problem, optimized over c_i, Y, and g_i, with some income level M_i and some set of prices p_c, p_Y, and p_g, that would produce the same solution. As discussed above, Figure A1 illustrates the relevant choice set facing an individual. Extending the ABCD trapezoid to the axes creates a hypothetical "budget plane" with corresponding hypothetical income and prices. Using points A, B, C, we have the system of equations:

$$\begin{pmatrix} w_i(1-\theta) & Y_{-i} & 0 & -1 \\ w_i(1-\tau) & Y_{-i} + \tau w_i & 0 & -1 \\ 0 & Y_{-i} + w_i(1-\theta) & w_i(1-\theta) & -1 \end{pmatrix} \begin{pmatrix} p_c \\ p_Y \\ p_g \\ M \end{pmatrix} = 0 \tag{15}$$

Where 0 is a 3×1 column of zeroes.[33] The system is solved by $p_c = \tau$, $p_g = \theta$, $p_Y = \tau - \theta$, and $M = Y_{-i}(\tau - \theta) + w_i(1-\theta)\tau$;[34] Figure A2 presents these income and price parameters.[35]

We could imagine an individual solving a three-variable problem facing these prices and income values:

$$\max_{c_i, Y, g_i} \; U^i(c_i,\ Y,\ g_i) + \lambda(M_i - p_c c_i - p_Y Y - p_g g_i). \tag{16}$$

Let the solutions to (16) be $\tilde{c}^*_i(M_i, p_c, p_Y, p_g)$, $\tilde{Y}^*(M_i, p_c, p_Y, p_g)$, and $\tilde{g}^*_i(M_i, p_c, p_Y, p_g)$, where the parameters M_i, p_c, p_Y, and p_g are functions of w_i, τ, θ, and Y_{-i}. Given convex preferences, an individual at an interior solution when solving the two-variable problem in (1) would reach the same solution when solving (16). Thus before and after any changes in the parameters w_i, τ, θ, and Y_{-i}, it must true that $Y^* = f^i(w_i, Y_{-i}, \tau, \theta) = \tilde{Y}^*(M_i, p_c, p_Y, p_g)$ and similarly for the best-response choices c^*_i and g^*_i. Thus the partial derivative f_θ can be expressed as: $f_\theta(w_i, Y_{-i}, \tau, \theta) = \frac{\partial \tilde{Y}^*}{\partial M}\frac{\partial M}{\partial \theta} + \frac{\partial \tilde{Y}^*}{\partial p_c}\frac{\partial p_c}{\partial \theta} + \frac{\partial \tilde{Y}^*}{\partial p_Y}\frac{\partial p_Y}{\partial \theta} + \frac{\partial \tilde{Y}^*}{\partial p_g}\frac{\partial p_g}{\partial \theta}$ (cf. equation (6) in Cornes and Sandler, 1994). These partial derivatives with respect to θ can be easily evaluated using the expressions in

[33] Since the equations in (15) describe a plane in $\{c_i, Y, g_i\}$ space, adding an additional equation (such as an equation for point D) would be superfluous and would not change the rank of the leftmost matrix. As is standard in consumer optimization problems, these income and price values are identified up to a constant (ie., the budget plane could be depicted by multiplying income and prices by a constant); this is unimportant for the analysis here.

[34] Note that one could use the solution for the hypothetical income level M to re-derive equation (14), thus providing an alternative proof to Proposition 2.

[35] Some of the values can be inferred from inspection of the figure directly, although note that line AC is not parallel to line EG in the figure.

Figure A2; this fact and noting that $Y^* = \tilde{Y}^*(M_i, p_c, p_Y, p_g)$ yields equation (8). The derivation of f_τ is done similarly.

Proof of Part (a) of Proposition 3

As shown in section 3D, $\frac{dR}{dg_{NGO}} = -\tau n \frac{\partial s_i}{\partial Y_{-i}} \left(1 - (n-1)\left(f_{Y_{-i}} - 1\right)\right)^{-1}$. By assumption, $0 < f_{Y_{-i}} < 1$, so the term in parentheses is positive. Thus the entire expression for $\frac{dR}{dg_{NGO}}$ is negative if $\frac{\partial s_i}{\partial Y_{-i}}$ is positive. To sign $\frac{\partial s_i}{\partial Y_{-i}}$, we focus on $\frac{\partial g_i^*}{\partial Y_{-i}}$. As before we can write $g_i = Y - Y_{-i} - (w - s_i)\tau$, so $\frac{\partial g_i^*}{\partial Y_{-i}} = f_{Y_{-i}} - 1 + \tau \frac{\partial s_i}{\partial Y_{-i}}$. Since we assume $f_{Y_{-i}} < 1$, $\frac{\partial g_i^*}{\partial Y_{-i}}$ can be positive only if $\frac{\partial s_i}{\partial Y_{-i}} > 0$.

Following the proof of Proposition 2, we will show that $\frac{\partial g_i^*}{\partial Y_{-i}} = \frac{\partial g_i^*}{\partial w_i}\left(\frac{\tau-\theta}{\tau-\tau\theta}\right)$. The term in parentheses is positive and by normality $\frac{\partial g_i^*}{\partial w_i} > 0$; it then follows that $\frac{\partial g_i^*}{\partial Y_{-i}}$ is positive, which implies that $\frac{\partial s_i}{\partial Y_{-i}}$ is positive, establishing the result.

We can rewrite $\frac{\partial g_i^*}{\partial Y_{-i}} = \frac{\partial g_i^*}{\partial w_i}\left(\frac{\tau-\theta}{\tau-\tau\theta}\right)$ as $f_{Y_{-i}} + \tau \frac{\partial s_i}{\partial Y_{-i}} - 1 = \left(f_{w_i} + \tau \frac{\partial s_i}{\partial w_i} - \tau\right)\left(\frac{\tau-\theta}{\tau-\tau\theta}\right)$, where we have again used the equation $g_i = Y - (w - s_i)\tau - Y_{-i}$. The proof of Proposition 2 establishes that $f_{Y_{-i}} = f_{w_i}\left(\frac{\tau-\theta}{\tau-\tau\theta}\right)$; these terms drop out, leaving $\tau \frac{\partial s_i}{\partial Y_{-i}} - 1 = \left(\tau \frac{\partial s_i}{\partial w_i} - \tau\right)\left(\frac{\tau-\theta}{\tau-\tau\theta}\right)$. Factoring out the τ on the right-hand side and multiplying through by $(1-\theta)$ leaves

$$(1-\theta)\left(\tau \frac{\partial s_i}{\partial Y_{-i}} - 1\right) = \left(\frac{\partial s_i}{\partial w_i} - 1\right)(\tau - \theta). \tag{17}$$

Following the steps in the proof of Proposition 2, we can write

$$\frac{\partial s_i}{\partial Y_{-i}} = \frac{\alpha_2 b_1 - \alpha_1 b_2}{a_1\alpha_2 - a_2\alpha_1}$$

$$\frac{\partial s_i}{\partial w_i} = \frac{\alpha_2 b_3 - \alpha_1 b_4}{a_1\alpha_2 - a_2\alpha_1}$$

where these terms are defined as in the proof of Proposition 2. Plugging these into equation (17) produces

$$(1-\theta)\left(\frac{\tau\alpha_2 b_1 - \tau\alpha_1 b_2 - a_1\alpha_2 + a_2\alpha_1}{a_1\alpha_2 - a_2\alpha_1}\right) = \left(\frac{\alpha_2 b_3 - \alpha_1 b_4 - a_1\alpha_2 + a_2\alpha_1}{a_1\alpha_2 - a_2\alpha_1}\right)(\tau - \theta). \tag{18}$$

The denominators are the same. Focusing first on the numerator of the left-hand side we can rewrite

$(1-\theta)\left(\tau\alpha_2 b_1 - \tau\alpha_1 b_2 - a_1\alpha_2 + a_2\alpha_1\right)$ as: $(1-\theta)\left(\alpha_2\left(\tau b_1 - a_1\right) + \alpha_1\left(a_2 - \tau b_2\right)\right)$. This expands to

$$
\begin{aligned}
&(1-\theta)\big[\big(-(\tau+\theta)U^i_{c_i g_i} + \tau U^i_{g_i Y} + \tau U^i_{g_i g_i} + \theta U^i_{c_i c_i} - \theta U^i_{c_i Y}\big) \\
&\times \big(\tau(\tau-\theta)U^i_{c_i g_i} - \tau(\tau-\theta)U^i_{g_i g_i} - \tau\theta U^i_{c_i Y} + \tau\theta U^i_{g_i Y} + \theta^2 U^i_{c_i Y} - \theta\tau U^i_{g_i Y} - \theta(\tau-\theta)U^i_{c_i g_i} + \tau(\tau-\theta)U^i_{g_i g_i}\big) \\
&+ \big(-\theta U^i_{c_i Y} + \theta U^i_{YY} + \theta U^i_{g_i Y} + (\tau-\theta)U^i_{c_i g_i} - (\tau-\theta)U^i_{g_i Y} - (\tau-\theta)U^i_{g_i g_i}\big) \\
&\times \big(-2\theta\tau U^i_{c_i g_i} + \tau^2 U^i_{g_i g_i} + \theta^2 U^i_{c_i c_i} - \tau\theta U^i_{c_i c_i} + \tau(\tau+\theta)U^i_{c_i g_i} - \tau^2 U^i_{g_i g_i}\big)\big]
\end{aligned}
$$

were again we exploit that $U_{\ell k} = U_{k\ell}$ and alphabetize subscripts. The $U^i_{g_i g_i}$ and $U^i_{g_i Y}$ terms cancel in the second set of parentheses, and the $U^i_{g_i g_i}$ terms cancel in the last set of parentheses. Gathering terms, we can write this as

$$
\begin{aligned}
&(1-\theta)(\tau-\theta)\big[\big(-(\tau+\theta)U^i_{c_i g_i} + \tau U^i_{g_i Y} + \tau U^i_{g_i g_i} + \theta U^i_{c_i c_i} - \theta U^i_{c_i Y}\big)\big((\tau-\theta)U^i_{c_i g_i} - \theta U^i_{c_i Y}\big) \\
&+ \big(-\theta U^i_{c_i Y} + \theta U^i_{YY} + (2\theta-\tau)U^i_{g_i Y} + (\tau-\theta)U^i_{c_i g_i} - (\tau-\theta)U^i_{g_i g_i}\big)\big(\tau U^i_{c_i g_i} - \theta U^i_{c_i c_i}\big)\big]
\end{aligned}
$$

Multiplying out and canceling terms, the part in brackets becomes

$$
\begin{aligned}
&\theta\tau U_{g_i Y}U_{c_i g_i} - \theta\tau U_{g_i Y}U_{c_i Y} - \theta(\tau-\theta)U^2_{c_i g_i} + (2\theta^2 - \theta\tau)U_{c_i g_i}U_{c_i Y} - \tau\theta U_{c_i Y}U_{g_i g_i} \\
&+ \theta^2 U^2_{c_i Y} + \tau\theta U_{YY}U_{c_i g_i} - \theta^2 U_{YY}U_{c_i c_i} - \theta(2\theta-\tau)U_{g_i Y}U_{c_i c_i} + \theta(\tau-\theta)U_{c_i c_i}U_{g_i g_i}. \qquad (19)
\end{aligned}
$$

The numerator on the right-hand side of (18) can be written $(\tau-\theta)\left(\alpha_2\left(b_3 - a_1\right) + \alpha_1\left(a_2 - b_4\right)\right)$.This expands to

$$
\begin{aligned}
&(\tau-\theta)\big[\big(-\tau U^i_{c_i g_i} + \tau U^i_{g_i Y} + \tau U^i_{g_i g_i} + \theta U^i_{c_i c_i} - \theta U^i_{c_i Y} - \theta U^i_{c_i g_i}\big) \\
&\times \big((\tau-\theta)U^i_{c_i g_i} - \tau(\tau-\theta)U^i_{g_i g_i} - \theta U^i_{c_i Y} + \theta\tau U^i_{g_i Y} + \theta^2 U^i_{c_i Y} - \theta\tau U^i_{g_i Y} - \theta(\tau-\theta)U^i_{c_i g_i} + \tau(\tau-\theta)U^i_{g_i g_i}\big) \\
&+ \big(-\theta U^i_{c_i Y} + \theta U^i_{YY} + (2\theta-\tau)U^i_{g_i Y} + (\tau-\theta)U^i_{c_i g_i} - (\tau-\theta)U^i_{g_i g_i}\big) \\
&\times \big(-2\theta\tau U^i_{c_i g_i} + \tau^2 U^i_{g_i g_i} + \theta^2 U^i_{c_i c_i} - \theta U^i_{c_i c_i} + \tau(1+\theta)U^i_{c_i g_i} - \tau^2 U^i_{g_i g_i}\big)\big].
\end{aligned}
$$

The $U^i_{g_i g_i}$ and $U^i_{g_i Y}$ terms cancel in the second set of parentheses, and the $U^i_{g_i g_i}$ terms again cancel in the last set of parentheses. We can write this as

$$
\begin{aligned}
&(\tau-\theta)(1-\theta)\big[\big(-(\tau+\theta)U^i_{c_i g_i} + \tau U^i_{g_i Y} + \tau U^i_{g_i g_i} + \theta U^i_{c_i c_i} - \theta U^i_{c_i Y}\big)\big((\tau-\theta)U^i_{c_i g_i} - \theta U^i_{c_i Y}\big) \\
&+ \big(-\theta U^i_{c_i Y} + \theta U^i_{YY} + (2\theta-\tau)U^i_{g_i Y} + (\tau-\theta)U^i_{c_i g_i} - (\tau-\theta)U^i_{g_i g_i}\big)\big(\tau U^i_{c_i g_i} - \theta U^i_{c_i c_i}\big)\big].
\end{aligned}
$$

The terms outside of the brackets are the same as before. Multiplying out and canceling the terms in brackets yields the expression in (19). Thus, $\frac{\partial g_i^*}{\partial Y_{-i}} = \frac{\partial g_i^*}{\partial w_i}\left(\frac{\tau-\theta}{\tau-\tau\theta}\right)$, so $\frac{\partial g_i^*}{\partial Y_{-i}}$ and $\frac{\partial s_i}{\partial Y_{-i}}$ are positive, and $\frac{dR}{dg_{NGO}}$ is therefore negative.

Proof of Part (b) of Proposition 3

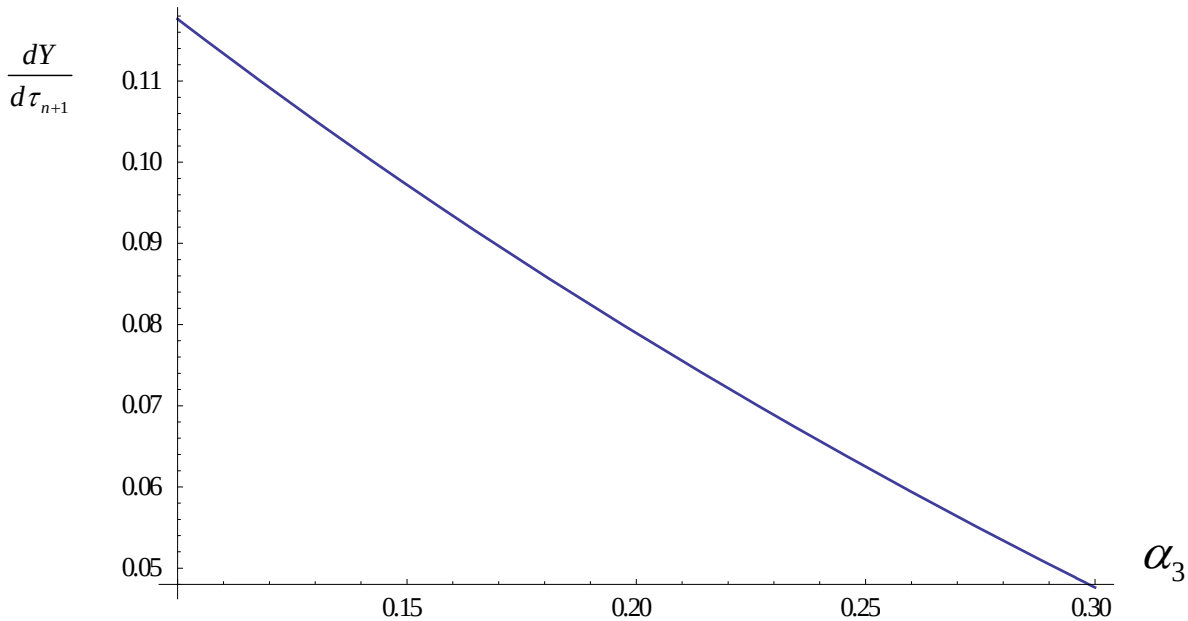
As in the proof for part (a), write $\frac{dR}{dg_{NGO}} = -\tau n \frac{\partial s_i}{\partial Y_{-i}}\left(1-(n-1)\left(f_{Y_{-i}}-1\right)\right)^{-1}$; call this term R_g. Following the proof of Proposition 1, we can write $f_{Y_{-i}} = 1 - \frac{\partial s_i}{\partial Y_{-i}}\theta - \frac{\partial c_i}{\partial Y_{-i}}$. Plugging this in yields $R_g = -\tau n \frac{\partial s_i}{\partial Y_{-i}}\left(1-(n-1)\left(1-\frac{\partial s_i}{\partial Y_{-i}}\theta - \frac{\partial c_i}{\partial Y_{-i}} - 1\right)\right)^{-1}$ or $R_g = -\tau n s_{Y_{-i}}\left(1+(n-1)\left(s_{Y_{-i}}\theta + \frac{\partial c_i}{\partial Y_{-i}}\right)\right)^{-1}$ where we let $s_{Y_{-i}} \equiv \frac{\partial s_i}{\partial Y_{-i}}$. By assumption $\frac{\partial c_i}{\partial Y_{-i}}$ is constant, so by equation (12) an increase in $\frac{\partial g_i}{\partial Y_{-i}}$ corresponds to an increase in $s_{Y_{-i}}$. All other terms are constants in R_g, and so the proof involves signing $\frac{\partial R_g}{\partial s_{Y_{-i}}}$. Straightforward differentiation shows

$$\frac{\partial R_g}{\partial s_{Y_{-i}}} = \frac{-\tau n\left(1+(n-1)\left(s_{Y_{-i}}\theta + \frac{\partial c_i}{\partial Y_{-i}}\right)\right) + \tau n s_{Y_{-i}}(n-1)\theta}{\left(1+(n-1)\left(s_{Y_{-i}}\theta + \frac{\partial c_i}{\partial Y_{-i}}\right)\right)^2}.$$

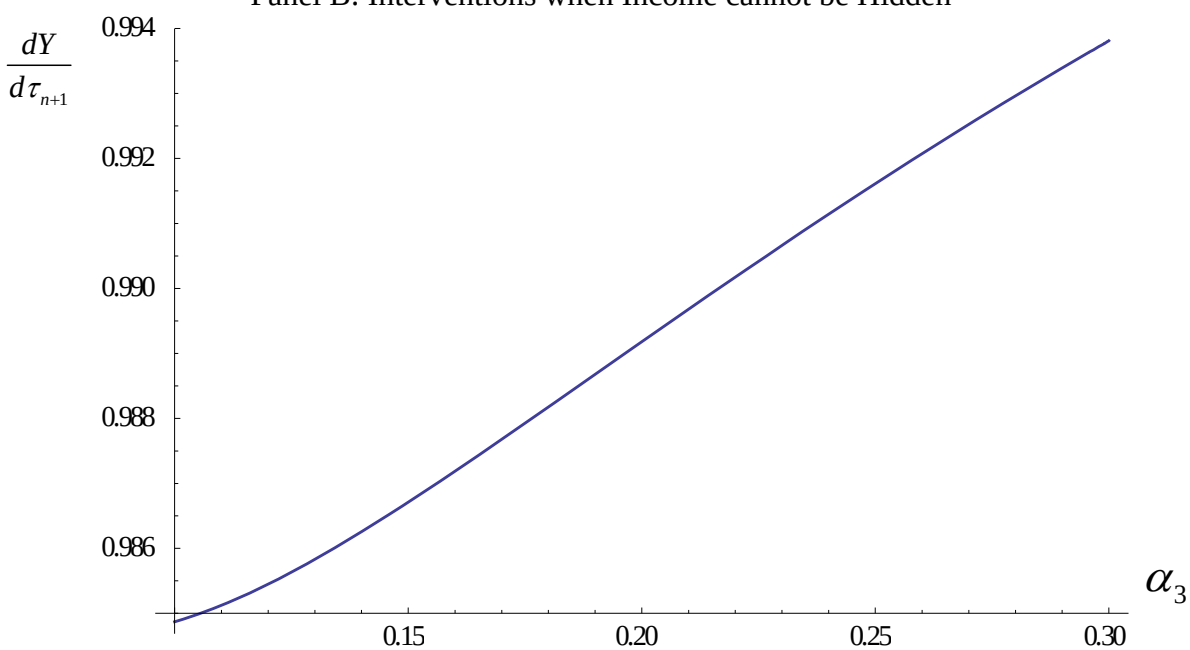
The denominator is positive and the numerator simplifies to $-\tau n\left(1+(n-1)\frac{\partial c_i}{\partial Y_{-i}}\right) < 0$. Then $\frac{\partial R_g}{\partial s_{Y_{-i}}} < 0$ and larger values of $\frac{\partial g_i}{\partial Y_{-i}}$ correspond to smaller (more negative) values of $\frac{dR}{dg_{NGO}}$.

Figure 1:
Public Good Provision and Strength of Warm Glow

Panel A: Interventions when Income is Hidden

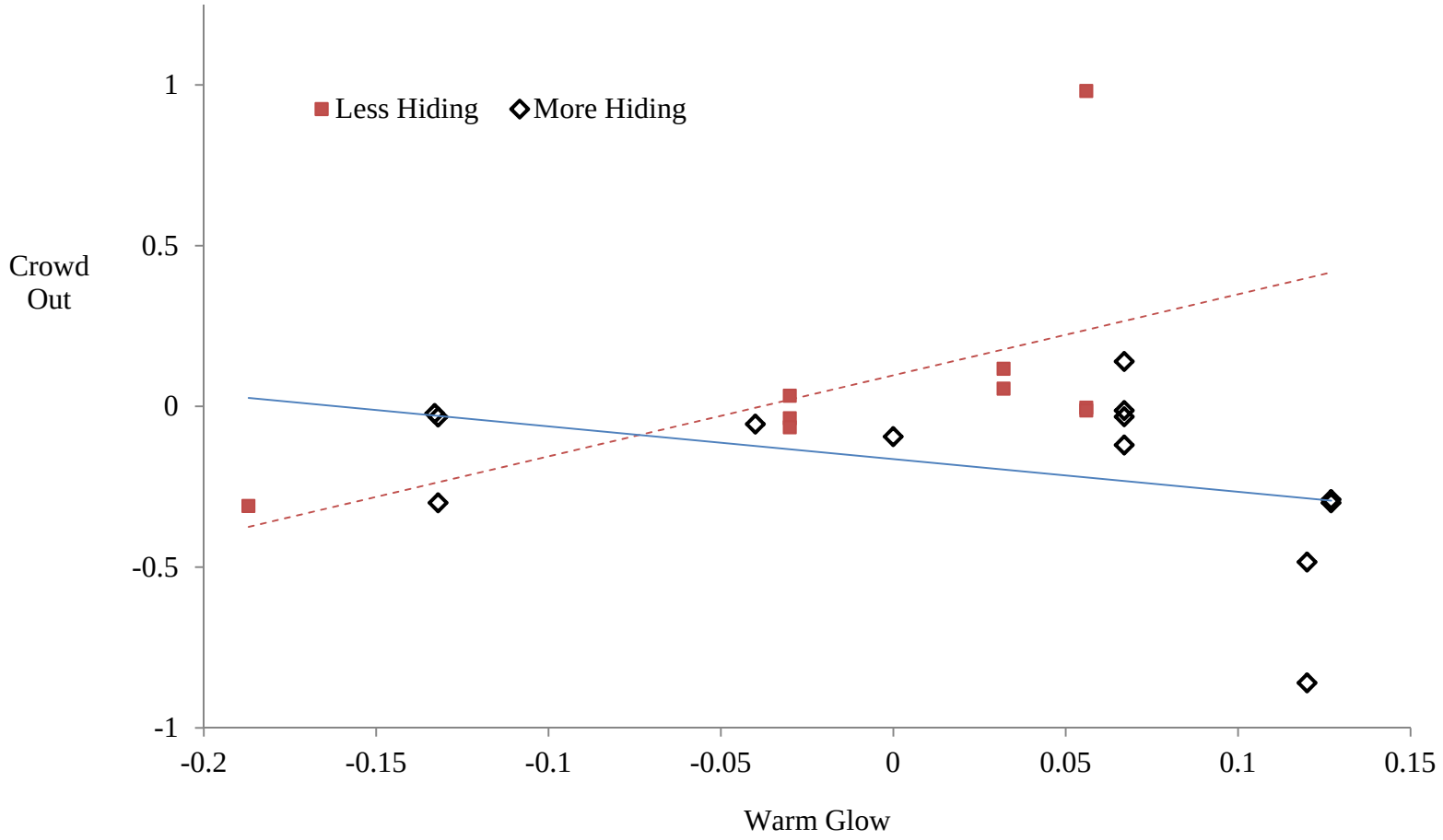


Panel B: Interventions when Income cannot be Hidden



The figure shows the effect of an exogenous increase in the public good in two symmetric Nash Equilibriums. In panel A, individuals hide some income in equilibrium; in panel B no income is allowed to be hidden. Larger values of α_3 correspond to stronger warm glow. Larger values of $dY/d\tau_{n+1}$ correspond to less crowd out and more effective policy intervention.

Figure 2:
Estimates of Crowd Out with More and Less Hidden Income



See Table 1 for the data used in this picture. The x-axis shows average warm glow in a country, and the y axis shows estimated crowd out of inter-family transfers (using the crowd out papers listed in Table 1). Countries with more than 30 percent of respondents stating it is sometimes justifiable to cheat on taxes are categorized in the "more hiding" group; although the picture is not critically sensitive to this cutoff.

Figure A1:
The Optimization Problem with Hidden Income

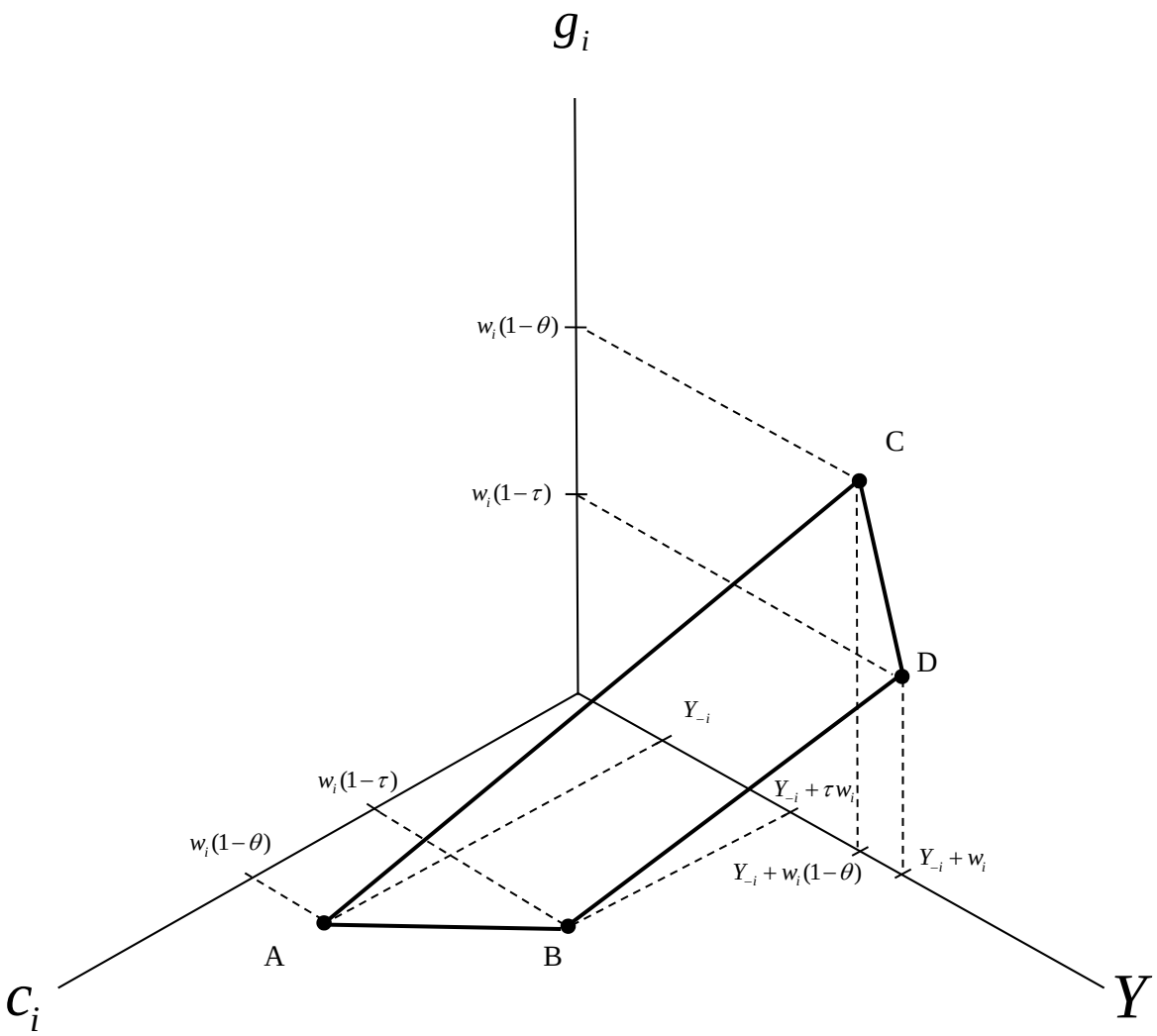


Figure A2:
Parameters in the Three-Variable Problem

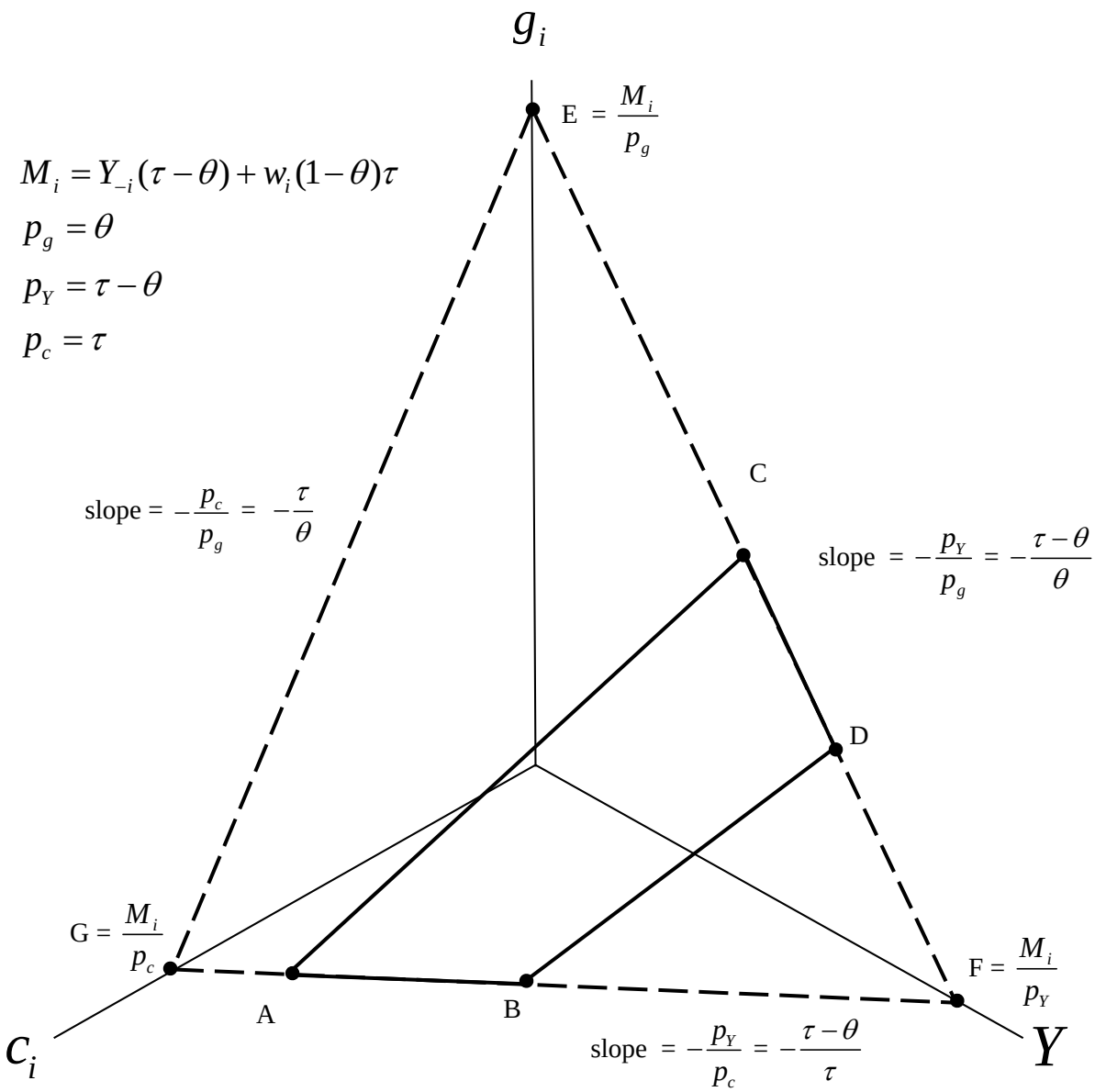


Table 1

Paper	Country	Cheating Justifiable?	Warm Glow	Crowd Out
McKernan, Pitt, and Moskowitz (2005)	Bangladesh	0.026	-0.187	-0.31
Gibson, Olivia, and Rozelle (2011)	Vietnam	0.18	0.032	0.117
Gibson, Olivia, and Rozelle (2011)	Vietnam	0.18	0.032	0.055
Raut and Tran (2005)	Indonesia	0.21	0.056	0.9811
Gibson, Olivia, and Rozelle (2011)	Indonesia	0.21	0.056	-0.004
Gibson, Olivia, and Rozelle (2011)	Indonesia	0.21	0.056	-0.013
Secondi (1997)	China	0.26	-0.03	0.033
Gibson, Olivia, and Rozelle (2011)	China	0.26	-0.03	-0.037
Cai, Giles, and Meng (2006)	China	0.26	-0.03	-0.065
Cox, Eser, and Jimenez (1998)	Peru	0.36	0.067	0.14
Cox and Jimenez (1992)	Peru	0.36	0.067	-0.12
Cox, Eser, and Jimenez (1998)	Peru	0.36	0.067	-0.032
Cox, Eser, and Jimenez (1998)	Peru	0.36	0.067	-0.013
Fan (2010)	Taiwan	0.37	0.127	-0.3
Lai and Orsuwan (2009)	Taiwan	0.37	0.127	-0.29
Jensen (2003)	South Africa	0.4	-0.132	-0.3
Maitra, Ray (2003)	South Africa	0.4	-0.132	-0.033
Kazianga (2006)	Burkina Faso	0.45	-0.04	-0.055
Cox, Jimenez, and Okrasa (1997)	Poland	0.47	0	-0.094
Albarran and Attanasio (2002)	Mexico	0.47	0.12	-0.484
Juarez (2009)	Mexico	0.47	0.12	-0.86
Cox, Hanson, Jimenez (2004)	Philippines	0.60	-0.133	-0.022

Column 3 shows the fraction of individuals in the World Values Survey in each country who say that cheating on your taxes is ever justifiable; the mean of this variable is 0.36. Column three shows the average warm glow in a country, using the regression coefficients described in the text. Prior to the regression, the mean of this variable is 0.19, std dev = 0.72 (the mean is zero following the regression; the reference country is Poland). All regressions and means use WVS weights. The cheating question is taken from the 1981-1984, 1989-1993, 1994-1999, 1999-2004, and 2005-2007 waves of the WVS with a sample of 46,464. The warm-glow measure excludes the 1981-1984 and 1994-1999 waves and has a sample of 24,628. Restricting the cheating measure to waves where warm glow is available produces similar means. The final column shows estimates of inter-family crowd out; the crowd-out estimates are from Table 15 & page 51 in McKernan et al (2005); Table 4 in Gibson et al (2011); Table 7 (summing the direct and indirect effects) in Raut and Tran (2005); Table 8 in Secondi (1997): Table 7 (using 3 times the poverty line, which is closest to average income) in Cai et al (2006), Table 1 in Cox et al (1998), the discussion on pages 166-167 (from Tables 2 and 3) in Cox et al (1992), Table 3 in Fan (2010), Table 4 in Lai and Orsuwan (2009), Table 4 (and discussion on page 105) in Jensen (2003), Table 6 in Maitra (2003), Table 9 (94/98 combined, both types of income) in Kazianga (2006), unnumbered table on page 204 of Cox, Jimenez, and Okrasa (1997), Table 7 (column 6, effect 1) from Alberran and Attanasio (2002), Table 4 and discussion on page 455 of Juarez (2009), and Table 3 of Cox et al (2004).