Introduction to Compilers and Language Design
Copyright © 2018 Douglas Thain.
First edition.

Anyone is free to download and print the PDF edition of this book for personal use. Commercial distribution, printing, or reproduction without the author’s consent is expressly prohibited. All other rights are reserved.

You can find the latest version of the PDF edition, and purchase inexpensive hardcover copies at http://compilerbook.org

Revision Date: February 12, 2019
Chapter 3 – Scanning

3.1 Kinds of Tokens

Scanning is the process of identifying tokens from the raw text source code of a program. At first glance, scanning might seem trivial – after all, identifying words in a natural language is as simple as looking for spaces between letters. However, identifying tokens in source code requires the language designer to clarify many fine details, so that it is clear what is permitted and what is not.

Most languages will have tokens in these categories:

- **Keywords** are words in the language structure itself, like `while` or `class` or `true`. Keywords must be chosen carefully to reflect the natural structure of the language, without interfering with the likely names of variables and other identifiers.

- **Identifiers** are the names of variables, functions, classes, and other code elements chosen by the programmer. Typically, identifiers are arbitrary sequences of letters and possibly numbers. Some languages require identifiers to be marked with a *sentinel* (like the dollar sign in Perl) to clearly distinguish identifiers from keywords.

- **Numbers** could be formatted as integers, or floating point values, or fractions, or in alternate bases such as binary, octal or hexadecimal. Each format should be clearly distinguished, so that the programmer does not confuse one with the other.

- **Strings** are literal character sequences that must be clearly distinguished from keywords or identifiers. Strings are typically quoted with single or double quotes, but also must have some facility for containing quotations, newlines, and unprintable characters.

- **Comments** and **whitespace** are used to format a program to make it visually clear, and in some cases (like Python) are significant to the structure of a program.

When designing a new language, or designing a compiler for an existing language, the first job is to state precisely what characters are permitted in each type of token. Initially, this could be done informally by stating,
token_t scan_token( FILE *fp ) {
    int c = fgetc(fp);
    if(c=='*') {
        return TOKEN_MULTIPLY;
    } else if(c=='!') {
        char d = fgetc(fp);
        if(d=='=') {
            return TOKEN_NOT_EQUAL;
        } else {
            ungetc(d,fp);
            return TOKEN_NOT;
        }
    } else if(isalpha(c)) {
        do {
            char d = fgetc(fp);
        } while(isalnum(d));
        ungetc(d,fp);
        return TOKEN_IDENTIFIER;
    } else if ( . . . ) {
        . . .
    }
}

Figure 3.1: A Simple Hand Made Scanner

for example, “An identifier consists of a letter followed by any number of letters and numerals.”, and then assigning a symbolic constant (TOKEN_IDENTIFIER) for that kind of token. As we will see, an informal approach is often ambiguous, and a more rigorous approach is needed.

3.2 A Hand-Made Scanner

Figure 3.1 shows how one might write a scanner by hand, using simple coding techniques. To keep things simple, we only consider just a few tokens: * for multiplication, ! for logical-not, != for not-equal, and sequences of letters and numbers for identifiers.

The basic approach is to read one character at a time from the input stream (fgetc(fp)) and then classify it. Some single-character tokens are easy: if the scanner reads a * character, it immediately returns TOKEN_MULTIPLY, and the same would be true for addition, subtraction, and so forth.

However, some characters are part of multiple tokens. If the scanner encounters !, that could represent a logical-not operation by itself, or it could be the first character in the != sequence representing not-equal-to. Upon reading !, the scanner must immediately read the next character. If
the next character is =, then it has matched the sequence != and returns \texttt{TOKEN NOT_EQUAL}.

But, if the character following ! is something else, then the non-matching character needs to be \texttt{put back} on the input stream using \texttt{ungetc}, because it is not part of the current token. The scanner returns \texttt{TOKEN NOT} and will consume the put-back character on the next call to \texttt{scan_token}.

In a similar way, once a letter has been identified by \texttt{isalpha(c)}, then the scanner keeps reading letters or numbers, until a non-matching character is found. The non-matching character is put back, and the scanner returns \texttt{TOKEN IDENTIFIER}.

(We will see this pattern come up in every stage of the compiler: an unexpected item doesn’t match the current objective, so it must be put back for later. This is known more generally as \texttt{backtracking}.)

As you can see, a hand-made scanner is rather verbose. As more token types are added, the code can become quite convoluted, particularly if tokens share common sequences of characters. It can also be difficult for a developer to be certain that the scanner code corresponds to the desired definition of each token, which can result in unexpected behavior on complex inputs. That said, for a small language with a limited number of tokens, a hand-made scanner can be an appropriate solution.

For a complex language with a large number of tokens, we need a more formalized approach to defining and scanning tokens. A formal approach will allow us to have a greater confidence that token definitions do not conflict and the scanner is implemented correctly. Further, a formalized approach will allow us to make the scanner compact and high performance – surprisingly, the scanner itself can be the performance bottleneck in a compiler, since every single character must be individually considered.

The formal tools of \texttt{regular expressions} and \texttt{finite automata} allow us to state very precisely what may appear in a given token type. Then, automated tools can process these definitions, find errors or ambiguities, and produce compact, high performance code.

### 3.3 Regular Expressions

Regular expressions (REs) are a language for expressing patterns. They were first described in the 1950s by Stephen Kleene \cite{kleene1950representations} as an element of his foundational work in automata theory and computability. Today, REs are found in slightly different forms in programming languages (Perl), standard libraries (PCRE), text editors (vi), command-line tools (grep), and many other places. We can use regular expressions as a compact and formal way of specifying the tokens accepted by the scanner of a compiler, and then automatically translate those expressions into working code. While easily explained, REs can be a bit tricky to use, and require some practice in order to achieve the desired results.
Let us define regular expressions precisely:

A regular expression \( s \) is a string which denotes \( L(s) \), a set of strings drawn from an alphabet \( \Sigma \). \( L(s) \) is known as the “language of \( s \).”

\( L(s) \) is defined inductively with the following base cases:

- If \( a \in \Sigma \) then \( a \) is a regular expression and \( L(a) = \{ a \} \).
- \( \epsilon \) is a regular expression and \( L(\epsilon) \) contains only the empty string.

Then, for any regular expressions \( s \) and \( t \):

1. \( s|t \) is a RE such that \( L(s|t) = L(s) \cup L(t) \).
2. \( st \) is a RE such that \( L(st) \) contains all strings formed by the concatenation of a string in \( L(s) \) followed by a string in \( L(t) \).
3. \( s^* \) is a RE such that \( L(s^*) = L(s) \) concatenated zero or more times.

Rule #3 is known as the Kleene closure and has the highest precedence. Rule #2 is known as concatenation. Rule #1 has the lowest precedence and is known as alternation. Parentheses can be added to adjust the order of operations in the usual way.

Here are a few examples using just the basic rules. (Note that a finite RE can indicate an infinite set.)

<table>
<thead>
<tr>
<th>Regular Expression s</th>
<th>Language L(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>hello</td>
<td>{ hello }</td>
</tr>
<tr>
<td>dog</td>
<td>ig</td>
</tr>
<tr>
<td>moo*</td>
<td>{ moo, moo, moo, ... }</td>
</tr>
<tr>
<td>(moo)*</td>
<td>{ moo, moo, moo, ... }</td>
</tr>
<tr>
<td>a(b</td>
<td>a)*a</td>
</tr>
</tbody>
</table>

The syntax described on the previous page is entirely sufficient to write any regular expression. But, it is also handy to have a few helper operations built on top of the basic syntax:

\( s? \) indicates that \( s \) is optional.
\( s? \) can be written as \((s|\epsilon)\)

\( s^+ \) indicates that \( s \) is repeated one or more times.
\( s^+ \) can be written as \( ss^* \)

\[a-z\] indicates any character in that range.
\[a-z\] can be written as \((a|b|...|z)\)

\[^x\] indicates any character except one.
\[^x\] can be written as \(\Sigma - x\)
Regular expressions also obey several algebraic properties, which make it possible to re-arrange them as needed for efficiency or clarity:

<table>
<thead>
<tr>
<th>Property</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associativity</td>
<td>( a \lor (b \lor c) = (a \lor b) \lor c )</td>
</tr>
<tr>
<td>Commutativity</td>
<td>( a \lor b = b \lor a )</td>
</tr>
<tr>
<td>Distribution</td>
<td>( a(b \lor c) = ab \lor ac )</td>
</tr>
<tr>
<td>Idempotency</td>
<td>( a^* = a^* )</td>
</tr>
</tbody>
</table>

Using regular expressions, we can precisely state what is permitted in a given token. Suppose we have a hypothetical programming language with the following informal definitions and regular expressions. For each token type, we show examples of strings that match (and do not match) the regular expression.

<table>
<thead>
<tr>
<th>Informal definition</th>
<th>Regular expression</th>
<th>Matches strings</th>
<th>Does not match</th>
</tr>
</thead>
<tbody>
<tr>
<td>An identifier is a sequence of capital letters and numbers, but a number must not come first.</td>
<td>([A-Z]+([A-Z]</td>
<td>[0-9])^*)</td>
<td>PRINT MODE5</td>
</tr>
<tr>
<td>A number is a sequence of digits with an optional decimal point. For clarity, the decimal point must have digits on both left and right sides.</td>
<td>([0-9]+(.[0-9]+)?)</td>
<td>123 3.14</td>
<td>.15 30.</td>
</tr>
<tr>
<td>A comment is any text (except a right angle bracket) surrounded by angle brackets.</td>
<td>(&lt;[^&gt;]*&gt;)</td>
<td>&lt;tricky part&gt; &lt;&lt;&lt;look left&gt;</td>
<td>&lt;this is an &lt;illegal&gt; comment&gt;</td>
</tr>
</tbody>
</table>

3.4 Finite Automata

A finite automaton (FA) is an abstract machine that can be used to represent certain forms of computation. Graphically, an FA consists of a number of states (represented by numbered circles) and a number of edges (represented by labelled arrows) between those states. Each edge is labelled with one or more symbols drawn from an alphabet \( \Sigma \).

The machine begins in a start state \( S_0 \). For each input symbol presented to the FA, it moves to the state indicated by the edge with the same label.
as the input symbol. Some states of the FA are known as accepting states and are indicated by a double circle. If the FA is in an accepting state after all input is consumed, then we say that the FA accepts the input. We say that the FA rejects the input string if it ends in a non-accepting state, or if there is no edge corresponding to the current input symbol.

Every RE can be written as an FA, and vice versa. For a simple regular expression, one can construct an FA by hand. For example, here is an FA for the keyword for:

![FA for the keyword for](image)

Here is an FA for identifiers of the form \[a-z][a-z0-9]+\):

![FA for identifiers](image)

And here is an FA for numbers of the form \((1-9)[0-9]*\) | 0\):

![FA for numbers](image)

### 3.4.1 Deterministic Finite Automata

Each of these three examples is a deterministic finite automaton (DFA). A DFA is a special case of an FA where every state has no more than one outgoing edge for a given symbol. Put another way, a DFA has no ambiguity: for every combination of state and input symbol, there is exactly one choice of what to do next.

Because of this property, a DFA is very easy to implement in software or hardware. One integer \(c\) is needed to keep track of the current state.
The transitions between states are represented by a matrix \((M[s, i])\) which encodes the next state, given the current state and input symbol. (If the transition is not allowed, we mark it with \(E\) to indicate an error.) For each symbol, we compute \(c = M[s, i]\) until all the input is consumed, or an error state is reached.

### 3.4.2 Nondeterministic Finite Automata

The alternative to a DFA is a **nondeterministic finite automaton** (NFA). An NFA is a perfectly valid FA, but it has an ambiguity that makes it somewhat more difficult to work with.

Consider the regular expression \([a-z]*ing\), which represents all lowercase words ending in the suffix \(ing\). It can be represented with the following automaton:

![Diagram of an automaton](image)

Now consider how this automaton would consume the word \(sing\). It could proceed in two different ways. One would be to move to state 0 on \(s\), state 1 on \(i\), state 2 on \(n\), and state 3 on \(g\). But the other, equally valid way would be to stay in state 0 the whole time, matching each letter to the \([a-z]\) transition. Both ways obey the transition rules, but one results in acceptance, while the other results in rejection.

The problem here is that state 0 allows for two different transitions on the symbol \(i\). One is to stay in state 0 matching \([a-z]\) and the other is to move to state 1 matching \(i\).

Moreover, there is no simple rule by which we can pick one path or another. If the input is \(sing\), the right solution is to proceed immediately from state zero to state one on \(i\). But if the input is \(singing\), then we should stay in state zero for the first \(ing\) and proceed to state one for the second \(ing\).

An NFA can also have an \(\epsilon\) (epsilon) transition, which represents the empty string. This transition can be taking without consuming any input symbols at all. For example, we could represent the regular expression \(a*(ab|ac)\) with this NFA:
This particular NFA presents a variety of ambiguous choices. From state zero, it could consume $a$ and stay in state zero. Or, it could take an $\epsilon$ to state one or state four, and then consume an $a$ either way.

There are two common ways to interpret this ambiguity:

- The **crystal ball interpretation** suggests that the NFA somehow “knows” what the best choice is, by some means external to the NFA itself. In the example above, the NFA would choose whether to proceed to state zero, one, or two before consuming the first character, and it would always make the right choice. Needless to say, this isn’t possible in a real implementation.

- The **many-worlds interpretation** suggests that that NFA exists in all allowable states simultaneously. When the input is complete, if any of those states are accepting states, then the NFA has accepted the input. This interpretation is more useful for constructing a working NFA, or converting it to a DFA.

Let us use the many-worlds interpretation on the example above. Suppose that the input string is $aaac$. Initially the NFA is in state zero. Without consuming any input, it could take an epsilon transition to states one or four. So, we can consider its initial state to be all of those states simultaneously. Continuing on, the NFA would traverse these states until accepting the complete string $aaac$:

<table>
<thead>
<tr>
<th>States</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 1, 4</td>
<td>consume $a$</td>
</tr>
<tr>
<td>0, 1, 2, 4, 5</td>
<td>consume $a$</td>
</tr>
<tr>
<td>0, 1, 2, 4, 5</td>
<td>consume $a$</td>
</tr>
<tr>
<td>0, 1, 2, 4, 5</td>
<td>consume $c$</td>
</tr>
<tr>
<td>6</td>
<td>accept</td>
</tr>
</tbody>
</table>

In principle, one can implement an NFA in software or hardware by simply keeping track of all of the possible states. But this is inefficient. In the worst case, we would need to evaluate all states for all characters on each input transition. A better approach is to convert the NFA into an equivalent DFA, as we show below.
3.5 Conversion Algorithms

Regular expressions and finite automata are all equally powerful. For every RE, there is an FA, and vice versa. However, a DFA is by far the most straightforward of the three to implement in software. In this section, we will show how to convert an RE into an NFA, then an NFA into a DFA, and then to optimize the size of the DFA.

<table>
<thead>
<tr>
<th>Regular Expression</th>
<th>Thompson’s Construction</th>
<th>Nondeterministic Finite Automaton</th>
<th>Subset Construction</th>
<th>Deterministic Finite Automaton</th>
<th>Transition Matrix</th>
<th>Code</th>
</tr>
</thead>
</table>

**Figure 3.2: Relationship Between REs, NFAs, and DFAs**

3.5.1 Converting REs to NFAs

To convert a regular expression to a nondeterministic finite automata, we can follow an algorithm given first by McNaughton and Yamada [5], and then by Ken Thompson [6].

We follow the same inductive definition of regular expression as given earlier. First, we define automata corresponding to the base cases of REs:

The NFA for any character $a$ is: The NFA for an $\epsilon$ transition is:

```
\[\quad \]
```

Now, suppose that we have already constructed NFAs for the regular expressions $A$ and $B$, indicated below by rectangles. Both $A$ and $B$ have a single start state (on the left) and accepting state (on the right). If we write the concatenation of $A$ and $B$ as $AB$, then the corresponding NFA is simply $A$ and $B$ connected by an $\epsilon$ transition. The start state of $A$ becomes the start state of the combination, and the accepting state of $B$ becomes the accepting state of the combination:

The NFA for the concatenation $AB$ is:
In a similar fashion, the alternation of $A$ and $B$ written as $A \mid B$ can be expressed as two automata joined by common starting and accepting nodes, all connected by $\epsilon$ transitions:

The NFA for the alternation $A \mid B$ is:

Finally, the Kleene closure $A^*$ is constructed by taking the automaton for $A$, adding starting and accepting nodes, then adding $\epsilon$ transitions to allow zero or more repetitions:

The NFA for the Kleene closure $A^*$ is:

Example. Let’s consider the process for an example regular expression $a(c a t \mid c o w)^*$. First, we start with the innermost expression $c a t$ and assemble it into three transitions resulting in an accepting state. Then, do the same thing for $c o w$, yielding these two FAs:

The alternation of the two expressions $c a t \mid c o w$ is accomplished by adding a new starting and accepting node, with epsilon transitions. (The boxes are not part of the graph, but simply highlight the previous graph components carried forward.)
Then, the Kleene closure \((\text{cat} | \text{cow})^*\) is accomplished by adding another starting and accepting state around the previous FA, with epsilon transitions between:

Finally, the concatenation of \(a(\text{cat} | \text{cow})^*\) is achieved by adding a single state at the beginning for \(a\): smaller pieces:

You can easily see that the NFA resulting from the construction algorithm, while correct, is quite complex and contains a large number of epsilon transitions. An NFA representing the tokens for a complete language could end up having thousands of states, which would be very impractical to implement. Instead, we can convert this NFA into an equivalent DFA.
3.5.2 Converting NFAs to DFAs

We can convert any NFA into an equivalent DFA using the technique of **subset construction**. The basic idea is to create a DFA such that each state in the DFA corresponds to multiple states in the NFA, according to the “many-worlds” interpretation.

Suppose that we begin with an NFA consisting of states $N$ and start state $N_0$. We wish to construct an equivalent DFA consisting of states $D$ and start state $D_0$. Each $D$ state will correspond to multiple $N$ states. First, we define a helper function known as the **epsilon closure**:

<table>
<thead>
<tr>
<th>Epsilon closure.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$−closure($n$) is the set of NFA states reachable from NFA state $n$ by zero or more $\epsilon$ transitions.</td>
</tr>
</tbody>
</table>

Now we define the subset construction algorithm. First, we create a start state $D_0$ corresponding to the $\epsilon$−closure($N_0$). Then, for each outgoing character $c$ from the states in $D_0$, we create a new state containing the epsilon closure of the states reachable by $c$. More precisely:

<table>
<thead>
<tr>
<th>Subset Construction Algorithm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given an NFA with states $N$ and start state $N_0$, create an equivalent DFA with states $D$ and start state $D_0$.</td>
</tr>
<tr>
<td>Let $D_0 = \epsilon$−closure($N_0$).</td>
</tr>
<tr>
<td>Add $D_0$ to a list.</td>
</tr>
<tr>
<td>While items remain on the list:</td>
</tr>
<tr>
<td>Let $d$ be the next DFA state removed from the list.</td>
</tr>
<tr>
<td>For each character $c$ in $\Sigma$:</td>
</tr>
<tr>
<td>Let $T$ contain all NFA states $N_k$ such that:</td>
</tr>
<tr>
<td>$N_j \in d$ and $N_j \xrightarrow{c} N_k$</td>
</tr>
<tr>
<td>Create new DFA state $D_i = \epsilon$−closure($T$)</td>
</tr>
<tr>
<td>If $D_i$ is not already in the list, add it to the end.</td>
</tr>
</tbody>
</table>

Figure 3.3: Subset Construction Algorithm
Example. Let’s work out the algorithm on the NFA in Figure 3.4. This is the same NFA corresponding to the RE \( a\{\text{cat} | \text{cow}\}^* \) with each of the states numbered for clarity.

1. Compute \( D_0 \) which is \( \epsilon\)-closure(\( N_0 \)). \( N_0 \) has no \( \epsilon \) transitions, so \( D_0 = \{N_0\} \). Add \( D_0 \) to the work list.

2. Remove \( D_0 \) from the work list. The character \( a \) is an outgoing transition from \( N_0 \) to \( N_1 \). \( \epsilon\)-closure(\( N_1 \)) = \{\( N_1, N_2, N_3, N_4, N_8, N_{13} \}\) so add all of those to new state \( D_1 \) and add \( D_1 \) to the work list.

3. Remove \( D_1 \) from the work list. We can see that \( N_4 \overset{c}{\rightarrow} N_5 \) and \( N_8 \overset{c}{\rightarrow} N_9 \), so we create a new state \( D_2 = \{N_5, N_9\} \) and add it to the work list.

4. Remove \( D_2 \) from the work list. Both \( a \) and \( o \) are possible transitions because of \( N_5 \overset{a}{\rightarrow} N_6 \) and \( N_9 \overset{a}{\rightarrow} N_{10} \). So, create a new state \( D_3 \) for the \( o \) transition to \( N_6 \) and new state \( D_5 \) for the \( a \) transition to \( N_{10} \). Add both \( D_3 \) and \( D_5 \) to the work list.

5. Remove \( D_3 \) from the work list. The only possible transition is \( N_6 \overset{w}{\rightarrow} N_7 \) so create a new state \( D_4 \) containing the \( \epsilon\)-closure(\( N_7 \)) and add it to the work list.

6. Remove \( D_5 \) from the work list. The only possible transition is \( N_{10} \overset{t}{\rightarrow} N_{11} \) so create a new state \( D_6 \) containing \( \epsilon\)-closure(\( N_{11} \)) and add it to the work list.
7. Remove $D_4$ from the work list, and observe that the only outgoing transition $c$ leads to states $N_5$ and $N_9$ which already exist as state $D_2$, so simply add a transition $D_4 \xrightarrow{c} D_2$.

8. Remove $D_6$ from the work list and, in a similar way, add $D_6 \xrightarrow{c} D_2$.

9. The work list is empty, so we are done.

3.5.3 **Minimizing DFAs**

The subset construction algorithm will definitely generate a valid DFA, but the DFA may possibly be very large (especially if we began with a complex NFA generated from an RE.) A large DFA will have a large transition matrix that will consume a lot of memory. If it doesn’t fit in L1 cache, the scanner could run very slowly. To address this problem, we can apply Hopcroft’s algorithm to shrink a DFA into a smaller (but equivalent) DFA.

The general approach of the algorithm is to optimistically group together all possibly-equivalent states $S$ into super-states $T$. Initially, we place all non-accepting $S$ states into super-state $T_0$ and accepting states into super-state $T_1$. Then, we examine the outgoing edges in each state $s \in T_i$. If, a given character $c$ has edges that begin in $T_i$ and end in different super-states, then we consider the super-state to be inconsistent with respect to $c$. (Consider an impermissible transition as if it were a transition to $T_E$, a super-state for errors.) The super-state must then be split into multiple states that are consistent with respect to $c$. Repeat this process for all super-states and all characters $c \in \Sigma$ until no more splits are required.

---

**DFA Minimization Algorithm.**

Given a DFA with states $S$, create an equivalent DFA with an equal or fewer number of states $T$.

First partition $S$ into $T$ such that:

- $T_0 =$ non-accepting states of $S$.
- $T_1 =$ accepting states of $S$.

Repeat:

- $\forall T_i \in T$:
  - $\forall c \in \Sigma$:
    - if $T_i \xrightarrow{c} \{ \text{more than one } T \text{ state} \}$,
      - then split $T_i$ into multiple $T$ states such that $c$ has the same action in each.

Until no more states are split.

---

Figure 3.5: Hopcroft’s DFA Minimization Algorithm
Example. Suppose we have the following non-optimized DFA and wish to reduce it to a smaller DFA:

```
Example. Suppose we have the following non-optimized DFA and wish to reduce it to a smaller DFA:
```

![DFA Diagram](image)

We begin by grouping all of non-accepting states 1, 2, 3, 4 into one super-state and the accepting state 5 into another super-state, like this:

```
We begin by grouping all of non-accepting states 1, 2, 3, 4 into one super-state and the accepting state 5 into another super-state, like this:
```

![Grouped States Diagram](image)

Now, we ask whether this graph is consistent with respect to all possible inputs, by referring back to the original DFA. For example, we observe that, if we are in super-state (1,2,3,4) then an input of \( a \) always goes to state 2, which keeps us within the super-state. So, this DFA is consistent with respect to \( a \). However, from super-state (1,2,3,4) an input of \( b \) can either stay within the super-state or go to super-state (5). So, the DFA is inconsistent with respect to \( b \).

To fix this, we try splitting out one of the inconsistent states (4) into a new super-state, taking the transitions with it:

```
To fix this, we try splitting out one of the inconsistent states (4) into a new super-state, taking the transitions with it:
```

![Split States Diagram](image)

Again, we examine each super-state for consistency with respect to each input character. Again, we observe that super-state 1,2,3 is consistent with respect to \( a \), but not consistent with respect to \( b \) because it can either lead to state 3 or state 4. We attempt to fix this by splitting out state 2 into its own super-state, yielding this DFA.
Again, we examine each super-state and observe that each possible input is consistent with respect to the super-state, and therefore we have the minimal DFA.

3.6 Limits of Finite Automata

Regular expressions and finite automata are powerful and effective at recognizing simple patterns in individual words or tokens, but they are not sufficient to analyze all of the structures in a problem. For example, could you use a finite automaton to match an arbitrary number of nested parentheses?

It’s not hard to write out an FA that could match, say, up to three pairs of nested parentheses, like this:

But the key word is *arbitrary*! To match any number of parentheses would require an infinite automaton, which is obviously impractical. Even if we were to apply some practical upper limit (say, 100 pairs) the automaton would still be impractically large when combined with all the other elements of a language that must be supported.

For example, a language like Python permits the nesting of parentheses () for precedence, curly brackets {} to represent object types, and square brackets [] to represent lists. An automaton to match up to 100 nested pairs of each in arbitrary order would have 1,000,000 states!

So, we limit ourselves to using regular expressions and finite automata for the narrow purpose of identifying the words and symbols within a problem. To understand the higher level structure of a program, we will instead use parsing techniques introduced in Chapter 4.

3.7 Using a Scanner Generator

Because a regular expression precisely describes all the allowable forms of a token, we can use a program to automatically transform a set of reg-
ular expressions into code for a scanner. Such a program is known as a scanner generator. The program Lex, developed at AT&T, was one of the earliest examples of a scanner generator. Vern Paxson translated Lex into the C language to create Flex, which is distributed under the Berkeley license and is widely used in Unix-like operating systems today to generate scanners implemented in C or C++.

To use Flex, we write a specification of the scanner that is a mixture of regular expressions, fragments of C code, and some specialized directives. The Flex program itself consumes the specification and produces regular C code that can then be compiled in the normal way.

Figure 3.6 gives the overall structure of a Flex file. The first section consists of arbitrary C code that will be placed at the beginning of scanner.c, like include files, type definitions, and similar things. Typically, this is used to include a file that contains the symbolic constants for tokens.

The second section states character classes, which are a symbolic shorthand for commonly used regular expressions. For example, you might declare DIGIT \[0-9\]. This class can be referred to later as \{DIGIT\}.

The third section is the most important part. It states a regular expression for each type of token that you wish to match, followed by a fragment of C code that will be executed whenever the expression is matched. In the simplest case, this code returns the type of the token, but it can also be used to extract token values, display errors, or anything else appropriate.

The fourth section is arbitrary C code that will go at the end of the scanner, typically for additional helper functions. A peculiar requirement of Flex is that we must define a function yywrap() which returns one to indicate that the input is complete at the end of the file. If we wanted to continue scanning in another file, then yywrap() would open the next file and return zero.

The regular expression language accepted by Flex is very similar to that of formal regular expressions discussed above. The main difference is that characters that have special meaning with a regular expression (like parenthesis, square brackets, and asterisk) must be escaped with a backslash or surrounded with double quotes. Also, a period (.) can be used to
match any character at all, which is helpful for catching error conditions.

Figure 3.7 shows a simple but complete example to get you started. This specification describes just a few tokens: a single character addition (which must be escaped with a backslash), the `while` keyword, an identifier consisting of one or more letters, and a number consisting of one or more digits. As is typical in a scanner, any other type of character is an error, and returns an explicit token type for that purpose.

Flex generates the scanner code, but not a complete program, so you must write a `main` function to go with it. Figure 3.8 shows a simple driver program that uses this scanner. First, the main program must declare as `extern` the symbols it expects to use in the generated scanner code: `yyin` is the file from which text will be read, `yylex` is the function that implements the scanner, and the array `yytext` contains the actual text of each token discovered.

Finally, we must have a consistent definition of the token types across the parts of the program, so into `token.h` we put an enumeration describing the new type `token_t`. This file is included in both `scanner.flex` and `main.c`.

Figure 3.10 shows how all the pieces come together. `scanner.flex` is converted into `scanner.c` by invoking `flex scanner.flex -o scanner.c`. Then, both `main.c` and `scanner.c` are compiled to produce object files, which are linked together to produce the complete program.

3.8 Practical Considerations

Handling keywords. - In many languages, keywords (such as `while` or `if`) would otherwise match the definitions of identifiers, unless specially handled. There are several solutions to this problem. One is to enter a regular expression for every single keyword into the Flex specification. (These must precede the definition of identifiers, since Flex will accept the first expression that matches.) Another is to maintain a single regular expression that matches all identifiers and keywords. The action associated with that rule can compare the token text with a separate list of keywords and return the appropriate type. Yet another approach is to treat all keywords and identifiers as a single token type, and allow the problem to be sorted out by the parser. (This is necessary in languages like PL/1, where identifiers can have the same names as keywords, and are distinguished by context.)

Tracking source locations. In later stages of the compiler, it is useful for the parser or typechecker to know exactly what line and column number a token was located at, usually to print out a helpful error message. (“Undefined symbol spider at line 153.”) This is easily done by having the scanner match newline characters, and increase the line count (but not return a token) each time one is found.

Cleaning tokens. Strings, characters, and similar token types need to
3.8. PRACTICAL CONSIDERATIONS

Contents of File: scanner.flex

```plaintext
{%
#include "token.h"
%
DIGIT [0-9]
LETTER [a-zA-Z]
%
(" "|\t|\n) /* skip whitespace */
\+ { return TOKEN_ADD; }
while { return TOKEN_WHILE; }
{LETTER}+ { return TOKEN_IDENT; }
{DIGIT}+ { return TOKEN_NUMBER; }
. { return TOKEN_ERROR; }
%
int yywrap() { return 1; }
%
Figure 3.7: Example Flex Specification

Contents of File: main.c

```plaintext
#include "token.h"
#include <stdio.h>

extern FILE *yyin;
extern int yylex();
extern char *yytext;

int main() {
    yyin = fopen("program.c","r");
    if(!yyin) {
        printf("could not open program.c!\n");
        return 1;
    }

    while(1) {
        token_t t = yylex();
        if(t==TOKEN_EOF) break;
        printf("token: %d text: %s\n",t,yytext);
    }
}
```

Figure 3.8: Example Main Program
Chapter 3. Scanning

Contents of File: token.h

typedef enum {
    TOKEN_EOF=0,
    TOKEN_WHILE,
    TOKEN_ADD,
    TOKEN_IDENT,
    TOKEN_NUMBER,
    TOKEN_ERROR
} token_t;

Figure 3.9: Example Token Enumeration

be cleaned up after they are matched. For example, "hello\n" needs to have its quotes removed and the backslash-n sequence converted to a literal newline character. Internally, the compiler only cares about the actual contents of the string. Typically, this is accomplished by writing a function string_clean in the postamble of the Flex specification. The function is invoked by the matching rule before returning the desired token type.

Constraining tokens. Although regular expressions can match tokens of arbitrary length, it does not follow that a compiler must be prepared to accept them. There would be little point to accepting a 1000-letter identifier, or an integer larger than the machine’s word size. The typical approach is to set the maximum token length (YYLMAX in flex) to a very large value, then examine the token to see if it exceeds a logical limit in the action that matches the token. This allows you to emit an error message that describes the offending token as needed.

Error Handling. The easiest approach to handling errors or invalid input is simply to print a message and exit the program. However, this is unhelpful to users of your compiler – if there are multiple errors, it’s (usually) better to see them all at once. A good approach is to match the
3.9 Exercises

1. Write regular expressions for the following entities. You may find it necessary to justify what is and is not allowed within each expression:

   (a) English days of the week: Monday, Tuesday, ...
   (b) All integers where every three digits are separated by commas for clarity, such as:
       78
       1,092
       692,098,000
   (c) Internet email addresses like "John Doe" <john.doe@gmail.com>
   (d) HTTP Uniform Resource Locators (URLs) as described by RFC-1738.

2. Write a regular expression for a string containing any number of X and single pairs of < > and { } which may be nested but not interleaved. For example these strings are allowed:
   XXX<XX(X)XXX>X
   X(X)X<X>X(X)X<X>X
   But these are not allowed:
   XXX<X<XX>>XX
   XX<XX{XX}XX>XX

3. Test the regular expressions you wrote in the previous two problems by translating them into your favorite programming language that has native support for regular expressions. (Perl and Python are two good choices.) Evaluate the correctness of your program by writing test cases that should (and should not) match.

4. Convert these REs into NFAs using Thompson’s construction:
   (a) for | [a-z]+ | [xb]?[0-9]+
   (b) a ( bc*d | ed ) d*
   (c) ( a*b | b*a | ba )*

5. Convert the NFAs in the previous problem into DFAs using the subset construction method.
6. Minimize the DFAs in the previous problem by using Hopcroft’s algorithm.

7. Write a hand-made scanner for JavaScript Object Notation (JSON) which is described at http://json.org. The program should read JSON on the input, and then print out the sequence of tokens observed: LBRACKET, STRING, COLON, etc... Find some large JSON documents online and test your scanner to see if it works.

8. Using Flex, write a scanner for the Java programming language. As above, read in Java source on the input and output token types. Test it out by applying it to a large open source project written in Java.
3.10 Further Reading


