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Examination of Karnis, Goldsmith & Mason Concentration Profiles

In this project we are examining the classic results of Karnis, Goldsmith & Mason (1966) who first showed that the velocity distribution for flow of a suspension through a tube was blunted from the expected parabolic distribution, but then via a tracer study found the concentration distribution to be uniform.

```
clear
```

Part 1: The experimental concentration distribution

The experimental concentration distribution may be determined by counting the number of spheres observed in each of four bins. We take the outer edge of each of the bin radii to be 1mm, 2mm, 3mm, and the outer edge to be 4mm. We do this for both the initial and final profiles. Because of the tracer approach, the standard deviation in the number of tracers is just the square root of the number observed (Poisson statistics). We convert to concentration, and calculate the 2 sigma error bars using the error propagation formula, taking the gradient numerically.

```
nf=[84 76 69 71]';
ninit=[80 76 72 71]';
r=[.1 .2 .3 .4]';

% We do initial and final values separately using the same code (cut and
% paste):
n=ninit;
phi=0.33*n*r(4)^2/(n(1)*r(1)^2+n(2)*(r(2)^2-r(1)^2)+n(3)*(r(3)^2-r(2)^2)+n(4)*(r(4)^2-r(3)^2));
% Note that the denominator is the same for all the bins, so we can get the
% array of concentrations in one shot: no loop required here.

varn=diag(n); %This is the matrix of covariance for Poisson statistics.
gradphi=zeros(length(phi),length(n)); %We initialize the gradient matrix.
nk=n;
for i=1:length(n)
    n(i)=n(i)+1;
    phit=0.33*n*r(4)^2/(n(1)*r(1)^2+n(2)*(r(2)^2-r(1)^2)+n(3)*(r(3)^2-r(2)^2)+n(4)*(r(4)^2-r(3)^2));
    gradphi(:,i)=phit-phi; %We get the ith column of the gradient matrix
    n=nk;
end

varphi=gradphi*varn*gradphi';
sigphi=diag(varphi).^5;

% Now we keep these values for later use:
sigphiinit=sigphi;
```

```

phiinit=phi;

% We do a little cut and paste to get the final concentrations!

n=nf;
phi=0.33*n*r(4)^2/(n(1)*r(1)^2+n(2)*(r(2)^2-r(1)^2)+n(3)*(r(3)^2-r(2)^2)+n(4)*(r(4)^2-r(3)^2));

varn=diag(n);
gradphi=zeros(length(phi),length(n));
nk=n;
for i=1:length(n)
    n(i)=n(i)+1;
    phit=0.33*n*r(4)^2/(n(1)*r(1)^2+n(2)*(r(2)^2-r(1)^2)+n(3)*(r(3)^2-r(2)^2)+n(4)*(r(4)^2-r(3)^2));
    gradphi(:,i)=phit-phi;
    n=nk;
end

varphi=gradphi*varn*gradphi';
sigphi=diag(varphi).^0.5;
sigphif=sigphi;
phif=phi;

% Now we plot this up:
figure(1)
plot(r-.05,phiinit,'or',r-.05,phif,'*g')
hold on
errorbar(r-0.05,phiinit,sigphiinit*2,'r')
errorbar(r-0.05,phif,sigphif*2,'g')
hold off
legend('Initial','Final')
xlabel('r')
ylabel('concentration')
title('Comparison of Initial and Final Concentration')
grid on

%And we produce the required table of concentrations with 2 sigma error:

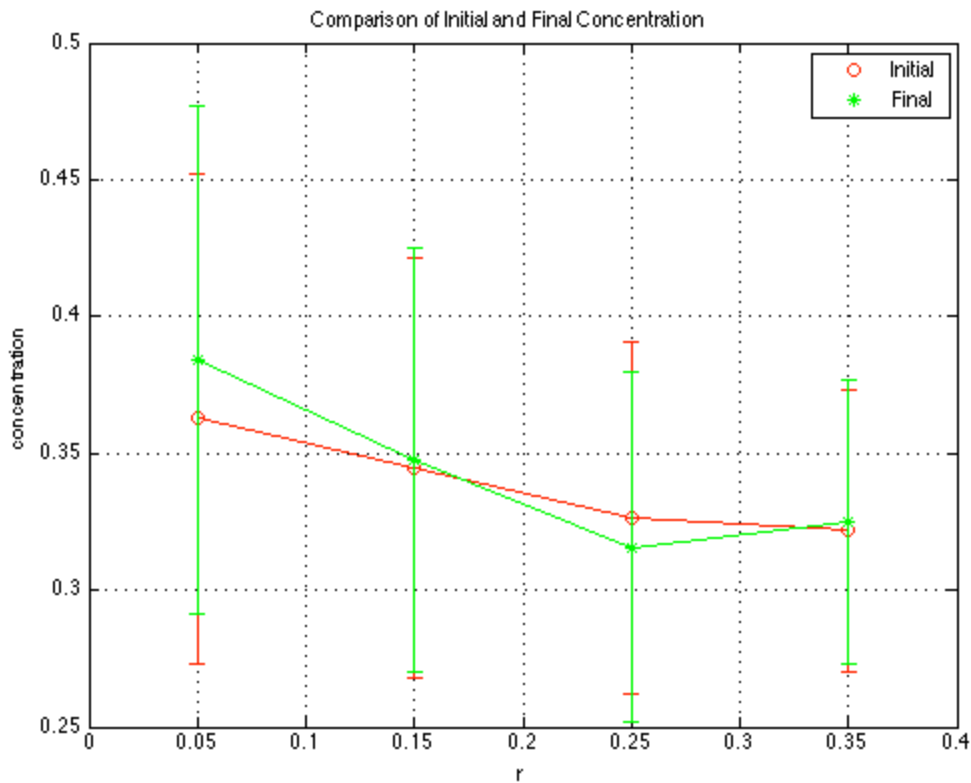
phitable=[r,phiinit,sigphiinit,phif,sigphif];
phitable=num2str(phitable);
top=str2mat('Initial & Final Concentrations with 2 sigma error',...
    ' r    initial phi  2 sigma    final phi    2 sigma');
phitable=str2mat(top,phitable);
disp(phitable)

```

```

Initial & Final Concentrations with 2 sigma error
 r    initial phi  2 sigma    final phi    2 sigma
0.1    0.36258    0.044661    0.38433    0.046428
0.2    0.34445    0.038317    0.34773    0.038654
0.3    0.32632    0.032085    0.3157     0.03191
0.4    0.32179    0.025845    0.32485    0.025924

```



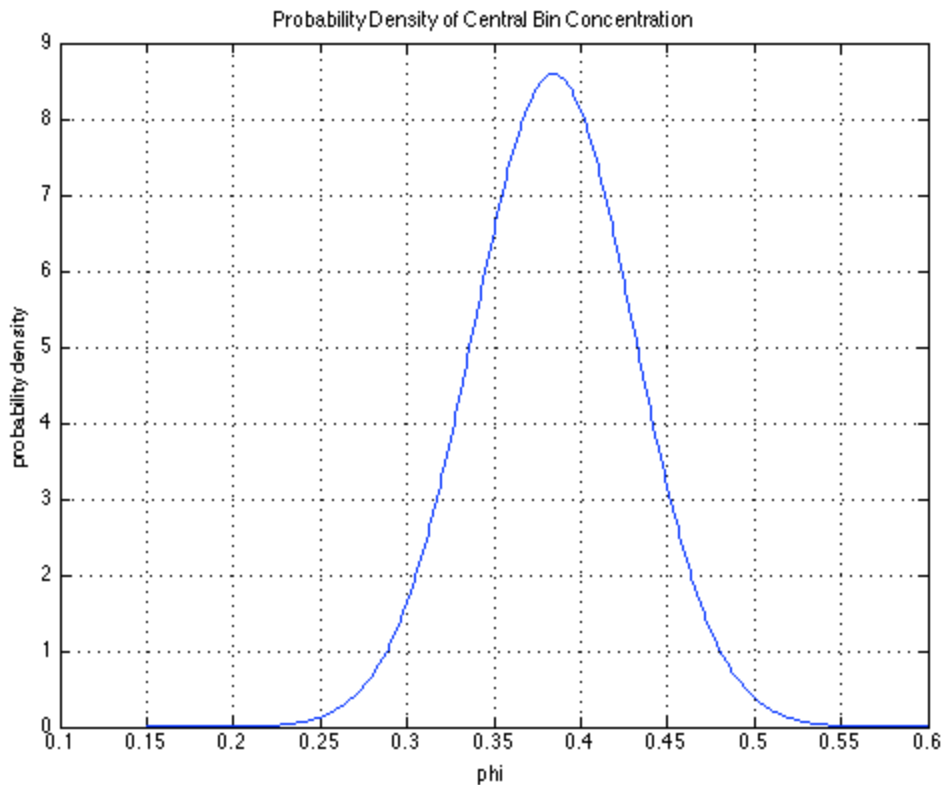
Part 2: The Central Bin Probability Distribution

We are particularly interested in the probability distribution of the central bin (closest to the center of the tube) at steady-state. We take this to be normally distributed with the calculated mean and standard deviation and plot it up:

```

phiplot=[.15:.001:.6]; %A range of concentrations for plotting purposes
probphi=pdf('normal',phiplot,phif(1),sigphif(1));
figure(2)
plot(phiplot,probphi)
xlabel('phi')
ylabel('probability density')
title('Probability Density of Central Bin Concentration')
grid on

```



Part 3: Comparison to Theoretical Models

The suspension balance model is probably the most robust predictor of the concentration distribution for flow in a tube. This yields the fully developed distribution with maximum packing in the center and a decrease as we move outward. We add it to our plot of concentration and compare it to the KG&M results. As can be seen, while the error bars on the experiments are huge, the theoretical values still lie outside the expected range.

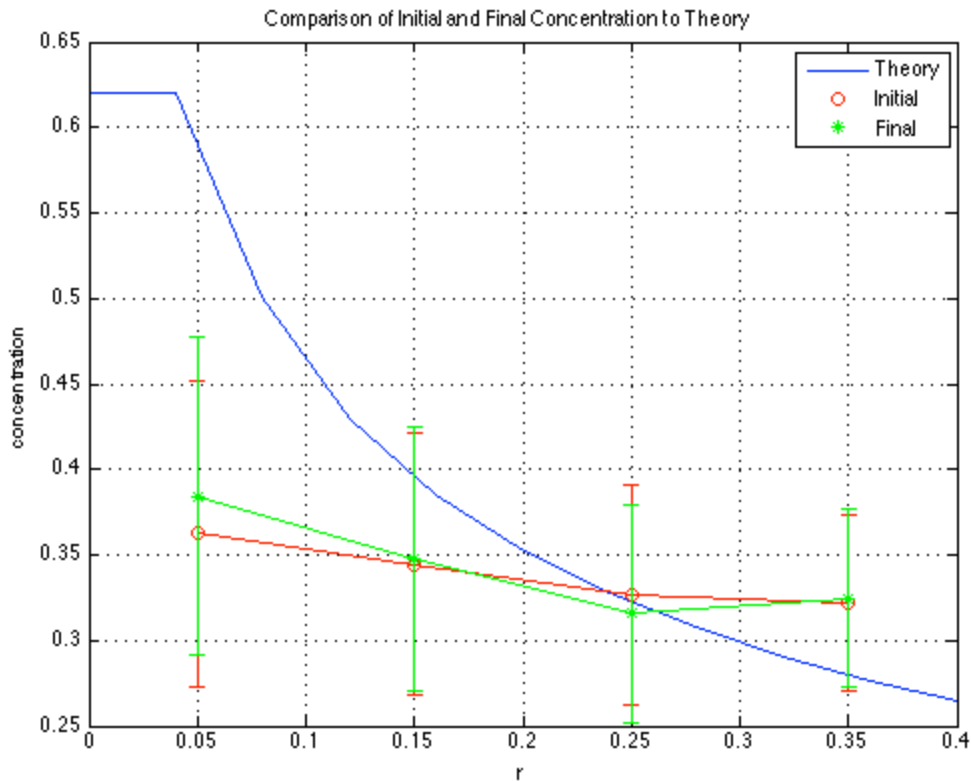
```
theory=[ 0.4      0.26484
0.36      0.27685
0.32      0.2908
0.28      0.30731
0.24      0.32729
 0.2      0.35225
0.16      0.3848
0.12      0.43011
0.08      0.50057
0.04      0.62
 0        0.62];
rtheory=theory(:,1);
phitheory=theory(:,2);

figure(3)
plot(rtheory,phitheory,r-.05,phiinit,'or',r-.05,phif,'*g')
hold on
errorbar(r-0.05,phiinit,sigphiinit*2,'r')
errorbar(r-0.05,phif,sigphif*2,'g')
hold off
legend('Theory','Initial','Final')
```

```

xlabel('r')
ylabel('concentration')
title('Comparison of Initial and Final Concentration to Theory')
grid on

```



Part 4: Statistical Comparison

We do two quantitative tests of the central bin concentrations. We compare to the expected value of 0.57, and the value of 0.45 necessary to cause significant blunting.

```

echo on
% First we do the 0.57 test:
zstat57=(phif(1)-0.57)/sigphif(1)
% so we are almost 4 sigma out. The probability of this occurring may be
% determined from the cdf:
prob57=cdf('normal',zstat57,0,1)
% which is pretty close to zero!

% Now we do the less stringent test:
zstat45=(phif(1)-0.45)/sigphif(1)
% and then we get the probability for this:
prob45=cdf('normal',zstat45,0,1)
% which is still pretty low, but much more reasonable.
echo off

```

```

% First we do the 0.57 test:
zstat57=(phif(1)-0.57)/sigphif(1)

```

```

zstat57 =
    -3.9990
% so we are almost 4 sigma out. The probability of this occurring may be
% determined from the cdf:
prob57=cdf('normal',zstat57,0,1)
prob57 =
    3.1803e-05
% which is pretty close to zero!
% Now we do the less stringent test:
zstat45=(phif(1)-0.45)/sigphif(1)
zstat45 =
    -1.4144
% and then we get the probability for this:
prob45=cdf('normal',zstat45,0,1)
prob45 =
    0.0786
% which is still pretty low, but much more reasonable.
echo off

```

The Conclusion:

Based on this analysis, there was a reasonable chance (about 1 in 13) that the central bin concentration was at least 45% based on random chance, but a vanishingly small probability that it was the much larger expected value of 57%. The question is why there was such a large deviation. As described in the problem statement, the most likely explanation was systematic bias: the system may not have been fully developed due to the length of the tube, and also that the particles may have been undercounted in the central bin due to optical issues. This is why it is always dangerous to draw conclusions solely from statistics: you are only getting information about random error in the measurements, and there may be systematic sources of error (which don't show up in the scatter) which are much larger...