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Cheg 258 First Hour Exam **3/4/98**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

Problem 1. Norms and Matrices:

a. Consider the vector $\underset{\approx}{x}$ = (-4, -4, 2). Calculate the norms:
1. 1-norm 2. 2-norm 3. ∞ - norm

b. Prove that the 2-norm condition number of an orthogonal matrix is unity.

c. Using the concept of singular value decomposition and the properties of orthogonal and diagonal matrices, prove that the 2-norm of the product $\underset{\approx}{A}^T \underset{\approx}{A}$ is the square of the 2-norm of $\underset{\approx}{A}$. Don't make this one too hard, because it's not...

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Problem 2. Systems of Equations:

In your laboratory you are doing protein separations. You are using UV absorbance to measure the protein concentration. For dilute solutions, the protein absorbance is proportional to the protein concentration, and the total absorbance is just the sum of that resulting from each species independently. Suppose you have calibrated the absorbance at three different wavelengths for three protein species:

	Absorbance measurements (units = 1/(g/liter))		
Species \ λ	200 nm	300 nm	400 nm
BSA	.1	.2	.1
BHb	.1	.2	.3
Insulin	.1	.3	.2

If you measure the absorbance at 200nm, 300nm, and 400nm to be 0.06, 0.15, and 0.11, respectively, solve for the concentration of each of the species.

Set this problem up as a 3x3 matrix problem of the form **A x** = **b** and **solve it using gaussian elimination**. Use the back of this sheet as well if you need more space.

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Problem 3. Linear Regression:

A rock is rapidly approaching a target, and we are interested in predicting the contact time t_f. If we use a quadratic model of the form:

$$\Delta x = C_1 (t_f - t) + C_2 (t_f - t)^2$$

we can fit a series of observed relative positions Δx_i at times t_i to determine the trajectory.

a. As posed, the model is non-linear in the parameter of interest t_f. Show how you can convert the problem to a linear form, clearly identifying the new parameters and modelling functions.

b. Set up the regression problem, clearly identifying the any matrices and variables you may use in terms of the original parameters C_1, C_2 and t_f and the arrays $\underset{\sim}{t}$ and $\underset{\sim}{\Delta x}$. Show how you can calculate t_f from your regression parameters.

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Problem 4. Error Propagation:

a. In the last problem you obtained an expression for the time for the rock to hit the target. Explicitly show how to calculate the error in the predicted contact time. **Be sure to list all of your assumptions!**

b. If we measure the components of a vector $\underset{\sim}{x}$ as $x_1 = 3 \pm .1$ and $x_2 = 4 \pm .2$ (error bars given are standard deviations), with a covariance between x_1 and x_2 of .01, calculate the 2-norm of the vector $\underset{\sim}{x}$ together with its one standard deviation error.

c. (one point extra credit) There are 55 students in the class, and I will ask approximately 80 questions of the day over the course of the semester. What is the probability that you will be called on at least once?