Cheg 258 Second Hour Exam **4/12/95**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

Problem 1. Statistics / Linear Regression:

A. (5 points) You are designing a weapon system which is supposed to hit a target moving with a constant velocity. The trick is to figure out how much to lead the target by determining where it will be in the future. The sighting system determines the location of the target at three independent times t = [1, 2, 3] milliseconds to be at positions [30, 39, 50] meters. We assume that the deviation in the target position from the expected linear relationship with time is due to scatter in the observations. Given these assumptions, show how you would estimate the position of the target at t = 4 milliseconds. Set this up as a **TWO PARAMETER** linear regression problem.

B. (10 points) Show how you would calculate the standard deviation of this estimate using the scatter in the observations themselves. Show all of your equations - I am far more interested in you getting the procedure correct than getting a numerical result. Only plug in the numbers to get the actual numerical result if you have enough time!

Problem 2. Non-linear root finding:

A. (10 points) Derive the secant method for finding roots to functions. Under what conditions does it fail to converge?

B. (5 points) Apply this method to the function $f(x) = x^2 - 4$ choosing as initial points $x =$ 0 and 1. Do two iterations.

Problem 3. Quadrature:

A. (10 points) We wish to develop a quadrature rule which integrates as high a degree polynomial over the domain [-1,1] as is possible. We pick the points to be evenly spaced at $x = [-1, -0.5, 0, 0.5, 1]$ rather than letting them be adjustable parameters. The node weights for this five point rule, however, are left arbitrary. Determine the optimum choice of node weights.

Hint: you will find the result similar to the problem you solved for homework on Monday.

B. (5 points) What is the polynomial degree of the resulting quadrature rule? How does it quantitatively compare to the degree of the five point Gaussian quadrature rule?

Problem 4. Non-Linear Minimization:

A. (4 points) What is the rate of convergence of the Golden Search for a minimum or maximum, and what is the value of C?

B. (4 points) For what conditions is this search procedure guaranteed to converge?

C. (7 points) Describe how you could use the penalty function approach to obtain a minimum of $F(x_1, x_2)$ if you have the inequality constraint $g(x_1, x_2) < 1$. Be specific.