**NAME** ______________________________

**Cheg 258 Third Hour Exam** **5/7/97**
**Closed Book and Closed Notes**

Problem 1). Integration Error Propagation:

a. Derive the local error and propagation error for the Backward Euler method.

b. What is the stability interval for this method?

c. Why or why not are Runge-Kutta methods useful for stiff ODE's?

**NAME** ______________________________

Problem 2). Adaptive Step Size Control:

a. The Euler method is a one stage Runge Kutta rule specified by:

$$k_1 = h\, f(t_n, y_n)$$

$$(y_{n+1})^{1s} = y_n + k_1$$

This can be extended to a two stage rule by adding the second stage:

$$k_2 = h\, f(t_n+h, y_n+k_1)$$

$$(y_{n+1})^{2s} = y_n + (k_1 + k_2)/2$$

Describe how you can use these two rules to determine the optimum step size to achieve a tolerance of $\tau$ in an adaptive integration scheme. Be as specific as possible (e.g., an equation is preferred). Remember that the error estimate will be dominated by the one stage rule!

b. Apply this method to determine the step size required for the first step in integrating the equation y' = - 4 y ; y(0) = 1 to a tolerance of 0.01.

**NAME** ______________________________

Problem 3). Error Propagation / Optimization:

A chocolatier can make either chocolate covered cherries, chocolate covered nuts, or a mixture of the two. He makes a greater profit (on average) for the cherries than for the nuts, but the market is a bit more volatile. In this problem we wish to determine the optimum fraction of cherries and nuts he should make. The average figures are as follows:

$$\mu_c = \$3.00 \;/\; \text{box} \qquad \sigma_c^2 = 1.25\ (\$ \;/\; \text{box})^2$$
$$\mu_n = \$2.50 \;/\; \text{box} \qquad \sigma_n^2 = 0.25\ (\$ \;/\; \text{box})^2$$
$$\sigma_{cn}^2 = 0.25\ (\$ \;/\; \text{box})^2$$

Note that there is some covariance between the profit on cherries and nuts. On the following I want equations and numbers, but words will yield partial credit!

a. Given that he makes a fraction x of chocolate covered cherries and (1-x) of nuts, calculate his expected profit per box and the standard deviation of that profit.

b. If he wishes to maximize his profit under adverse economic assumptions (e.g., he wishes to maximize his profit at the 85% ($1\sigma$) confidence level), what should be his strategy? You don't actually need a calculator to solve this, although it might help.

c. How does the strategy change if he averages the behavior over two years, assuming no covariance from year to year?

**NAME** ________________________________

Problem 4). Systems of Equations:

a. A damped linear oscillator is governed by the second order differential equation:

$$y'' + D\,y' + y = 0 \;\; ; \;\; y(0) = 1,\; y'(0) = 0$$

where D is the damping coefficient. Convert this equation into a system of first order differential equations and appropriate boundary conditions.

b. For what values of D is this system of equations stable?

**NAME** ______________________________

c. An application of the methods we have developed for ODE's is the solution of parabolic PDE's. In this part we look at the one dimensional time dependent transport problem given by:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \quad ; \quad T\big|_{t=0} = 1 \ , \ T\big|_{x=0,1} = 0$$

We wish to numerically integrate this in time as a system of ODE's. I want you to:

1. Discretize the domain in the x direction, e.g., let $\underset{\sim}{T} = (T_0, T_1, T_2, ..., T_n)$ where $T_0$ is the value of T at x = 0 and $T_n$ is the value at x = 1.

2. Use a finite difference representation for the spatial second derivative to get $\underset{\sim}{T}$ at each position in the interior of the domain.

3. Using the answer to part 2, develop a system of first order differential equations in time for each node location.

4. Finally, show how you could use the Euler method to update your solution vector $\underset{\sim}{T}$ in time. Try to do this in vector form, clearly defining all matrices.