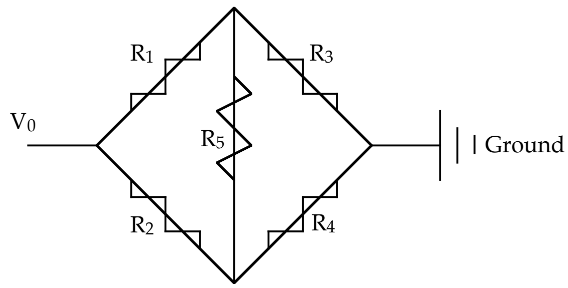
CBE 20258 Algorithm Problem Set 2

Due in class 2/4/16

Problem 1). A Wheatstone Bridge is an electrical circuit that is used in many chemical engineering sensor applications. As just one example, it forms the basis for solute detection in a gas chromatograph. It is depicted below:



A voltage V_0 is applied across the bridge, and the voltage differential between the two nodes in the middle is measured.

a. Using Ohm's Law V = I R, set up the system of five equations which the current through each of the five resistors must satisfy. Remember that current is conserved, and that voltage is a single value at any position. (e.g., voltage drop along any paths connecting the same two nodes must be identical)

b. Recast these equations in matrix form (e.g., Ax = b), clearly identifying A, b, and x.

Problem 2). Protein Assay: In your laboratory you are doing protein separations, and are using UV absorbance to measure protein concentrations. For dilute solutions, the protein absorbance is proportional to the concentration, and the total absorbance is just the sum of that resulting from each species independently. Suppose you have calibrated the absorbance at three different wavelengths for the three protein species:

Absorbance measurements (units = 1/(g/liter))

Species \ λ	200*nm*	300*nm*	400*nm*
BSA	0.1	0.2	0.1
BHb	0.1	0.2	0.3
Insulin	0.1	0.3	0.2

If you measure the absorbance at 200nm, 300nm, and 400nm to be 0.06, 0.15, and 0.11, respectively (dimensionless values):

a. Set up the problem for the concentration of each of the species in matrix form and

b. Solve for the concentrations.

Problem 3). Singular Value Decomposition. A non-square (nxm where n≠m) matrix $\underset{\approx}{A}$ undergoes SVD such that:

$$\underset{\approx}{A} = \underset{\approx}{U}\,\underset{\approx}{\Sigma}\,\underset{\approx}{V}^T$$

a. What are the dimensions of:

1) $\underset{\approx}{U}$

2) $\underset{\approx}{\Sigma}$

3) $\underset{\approx}{V}$

b. What is the inner product of the first and second columns of $\underset{\approx}{U}$ (e.g., U(: , 1)'*U(: , 2) in matlab)

c. What does matlab yield for the inner product of the first column of $\underset{\approx}{U}$ with the first column of $\underset{\approx}{V}$?

Problem 4) Singular Value Decompostion. A **square** matrix $\underset{\approx}{A} = \underset{\approx}{U}\,\underset{\approx}{\Sigma}\,\underset{\approx}{V}^T$ is decomposed as depicted below. All norms in this problem are taken to be **2-norms**.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix} = \begin{bmatrix} -0.2093 & 0.9644 & 0.1617 \\ -0.5038 & 0.0353 & -0.8631 \\ -0.8380 & -0.2621 & 0.4785 \end{bmatrix} \begin{bmatrix} 17.4125 & 0 & 0 \\ 0 & 0.8752 & 0 \\ 0 & 0 & 0.1969 \end{bmatrix} \begin{bmatrix} -0.4647 & -0.5538 & -0.6910 \\ -0.8333 & 0.0095 & 0.5528 \\ 0.2995 & -0.8326 & 0.4659 \end{bmatrix}$$

We are using this to solve the problem $\underset{\approx}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$

a. Suppose $\underset{\sim}{b} = \begin{bmatrix} 1.9288 \\ 0.0706 \\ -0.5243 \end{bmatrix}$ (e.g., twice the second column of $\underset{\approx}{U}$). Using SVD (rather than the matrix $\underset{\approx}{A}$ - show how you do this via orthogonality with pencil & paper!) what is the solution vector $\underset{\sim}{x}$?

b. If, for an arbitrary $\underset{\sim}{b}$ (not necessarily that in part a!), we have an error given by $\Delta\underset{\sim}{b}$, this will yield some error in $\underset{\sim}{x}$ given by $\Delta\underset{\sim}{x}$. What is the maximum of $\|\Delta\underset{\sim}{x}\| / \|\Delta\underset{\sim}{b}\|$?

c. In control, we often go the other way – there is some error in the output $\Delta\underset{\sim}{b}$ due to error in the input $\Delta\underset{\sim}{x}$. What is the maximum value of $\|\Delta\underset{\sim}{b}\| / \|\Delta\underset{\sim}{x}\|$?