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Cheg 258 First Hour Exam **2/26/97**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

Problem 1. Norms and Matrices:

a. Calculate the 1-norm condition numbers of the following matrices. The fourth matrix is orthogonal.

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2. $\begin{bmatrix} 4 & 0 \\ 0 & -0.2 \end{bmatrix}$ 3. $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ 4. $\begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & -\frac{3}{5} \end{bmatrix}$

b. Prove that the 2-norm of a matrix $\underset{\approx}{A}$ is the magnitude of its largest singular value (e.g., the largest element of the diagonal matrix produced in singular value decomposition). Note that I am not actually asking you to -do- SVD on the matrix, but you may assume that it has been done. State the properties of orthogonal matrices that you are using to complete your proof.

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Problem 2. Systems of Equations:

You are given raw feedstocks of chemicals A, B, and C. You are to combine these chemicals to form the products D, E, and F. The stoichiometry of the reactions are given below:

$$C + 2\,B \rightarrow D\,; \qquad B + 2\,A \rightarrow E\,; \qquad 2\,C + A \rightarrow F$$

If the amount of the three feedstocks are A = 1, B = 2, and C = 3, respectively, determine the amount of D, E, and F you can produce if all of the feedstock is to be consumed.

Set this problem up as a 3x3 matrix problem of the form **A x** = **b** and **solve it using gaussian elimination**. Use the back of this sheet as well if you need more space.

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Problem 3. Linear Regression:

A Meteorologist wants to determine the point (date) during the year in which the temperature is likely to be the coldest. All the data that is available is the average monthly temperature for the 12 months of one full year. It is proposed that the data be fitted by the cosine wave:

$$T = C \cos\left(\frac{2\,\pi\, t}{365} + \phi\right)$$

where T_i is the average temperature of a given month, and t_i is the time in days from the beginning of the year to the middle of the ith month. C is some constant characteristic of the amplitude of the seasonal temperature variation, and ϕ is the phase lag of interest - the point during the year when the temperature is either hottest or coldest.

a. As posed, the model is non-linear in the parameter of interest ϕ. Show how you can convert the problem to a linear form, clearly identifying the new parameters and modelling functions. You may find the trigonometric relation cos(a+b)=cos(a)cos(b)-sin(a)sin(b) to be of some use.

b. Set up the regression problem, clearly identifying the any matrices and variables you may use in terms of the original parameters C and ϕ and the arrays $\underset{\sim}{t}$ and $\underset{\sim}{T}$. Show how you can calculate ϕ from your regression parameters.

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Problem 4. Error Propagation:

In the last problem you obtained an expression for the coldest time of the year via linear regression of observed average monthly temperatures. In this problem we explore the error in this calculation.

a. Show how you calculate the error in your linear regression fitting parameters from the deviation between the best fit model and the measured temperature data. **What assumptions are you making here???**

b. In problem 3 you obtained an explicit expression $\phi = f(\underset{\sim}{x})$ where $\underset{\sim}{x}$ represents your regression parameters. Show how you can use this to calculate the error in ϕ from the error in $\underset{\sim}{x}$ (e.g., the matrix of covariance you obtained in part a). **What assumptions are you making here???**