Cheg 258 First Hour Exam **2/24/99**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

Problem 1. Finite Differences

In class we looked at the error resulting from the forward difference approximation to a derivative. In this problem we will look at the backward difference formula for a first derivative:

$$f'(x_0) \approx [f(x_0) - f(x_0 - h)] / h$$

a). What is the order in h of the algorithm error of this approximation?

b). Combining this algorithm error with numerical error in the calculation of f(x) determine the approximate optimum value of h and the minimum possible error. We are not worried about constants here, but I want to know how they depend on the machine error ε and the function and its derivatives.

Problem 2. Systems of Equations:

It is the end of the semester and you are about ready to pack up and go home. Looking around your kitchen you discover that you have four cups of butter, eight cups of flour, and four cups of sugar left over, as well as a pretty good selection of spices. While you are happy to take the spices home (they're pretty expensive these days!) you want to get rid of the rest of the ingredients by baking cookies for your friends. Looking through your copy of **The Joy of Cooking**, you find the following cookie recipes, with required ingredients:

Shortbread: 1 c butter, 2 c flour, $\frac{1}{2}$ c sugar
Roll cookies: $\frac{1}{2}$ c butter, $2\frac{1}{2}$ c flour, $\frac{1}{2}$ c sugar
Brandy snaps: $\frac{1}{2}$ c butter, 1 c flour, $\frac{3}{4}$ c sugar

You find that you have all the extra spices these recipes call for. How many recipes of each type of cookie do you need to make to exactly use up all the butter, flour, and sugar?

Set this problem up as a 3x3 matrix problem of the form **A x** = **b** and **solve it using gaussian elimination**. Use the back of this sheet as well if you need more space.

Problem 3. Linear Regression:

In senior lab many of you will measure the velocity profile resulting from a heated wire. Theory predicts that the centerline velocity of this flow field (closely related in shape, origin, and magnitude to the draft which rolls off a cold window) should vary as the height above the wire x to some power. We want to fit the data by the model:

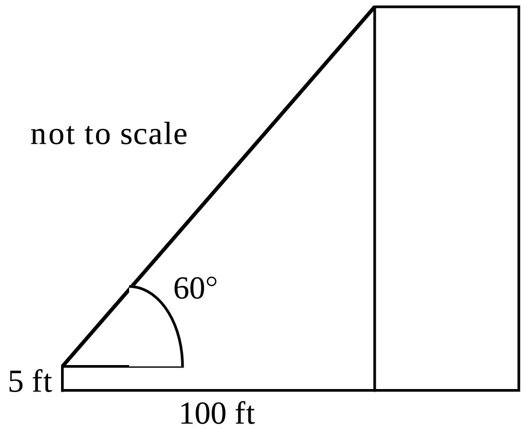
$$u = u_c \, x^{\lambda}$$

where u_c is some characteristic velocity and λ is the power law exponent we are interested in. Show how you can solve this problem using linear regression, clearly identifying any matrices and variables you may use in terms of the original parameters u_c and λ and the measurement arrays $\underset{\sim}{x}$ and $\underset{\sim}{u}$.

x (cm)	u (cm/s)
1	0.50
2	0.57
3	0.62
4	0.65
5	0.69

Problem 4. Error Propagation:

a. We are estimating the height of a building by using a transit, as is depicted in the diagram below. If the distance from the building to the transit is 100±3 ft, the transit itself is 5 ft above the base of the building (no error), and the angle on the transit when focussed on the roof edge is 60°±1° (all error estimates given are one standard deviation), calculate the estimated height of the building and its 95% confidence interval.



b. We often do quality control checks by selecting out samples at random from our product stream. I measure the following weights of candy bars selected at random. How many bars will I have to sample until the one standard deviation uncertainty in the mean weight is less than 1%? Be sure to show all formulas that you use.

3.0
2.8
3.2
3.1
2.9