CHEG 258 NUMERICAL METHODS

Final Exam
5/8/95

This test is closed books and closed notes

Problem 1. (15 points) Statistics:

The total amount of oil and natural gas consumed in the United States over the last 5 years has been measured. The values (in terms of energy equivalent, 10^{12} BTU) are given below. In this problem we are going to do a statistical study of this data.

Year	Nat Gas (x)	Oil (y)
1990	12.296	33.553
1991	19.606	32.845
1992	20.131	33.527
1993	20.841	33.841
1994	21.156	34.653

A. Write down the expressions for calculating the means, variances, and covariance of these variables.

B. Suppose we have a general function of these two variables z = f(x,y). Write down the expression for calculating the variance in z given that we have the variance and covariance of x and y. Why is this formula not exact?

C. Calculate the mean, variance, and covariance of the natural gas and oil usage, and use these values to estimate the variance i the average of z = (y/x - 1), the average relative excess of oil consumed over natural gas during the last five years.

Problem 2. (15 points) Integration:

Simultaneous heat and momentum transfer to an infinite flat plate is governed by the pair of coupled ordinary differential equations and boundary conditions given below:

$$f''' + 0.5\, f'\, f'' = 0$$
$$g'' = -0.5\, \Pr\, \eta\, f\, g'$$

with boundary conditions:

$$f(0) = f'(0) = 0; \;\; f'(\infty) = 1$$
$$g(0) = 0; \; g(\infty) = 1$$

where f is the dimensionless stream function, f ' is the velocity parallel to the plate, and g is the temperature. The variable η is the distance normal to the plate (the independent variable), with $\eta = 0$ being on the plate itself.

A). Rewrite the equation and boundary conditions as a system of first order equations.

B). Write out the Jacobian for this problem.

C). If you solve this problem using the shooting method, how many shooting parameters are there and what are they?

Problem 3. (15 points) Adaptive Integration:

The two stage Runge-Kutta rule is given by:

$$k_1 = h\, f(t_n, y_n)$$

$$k_2 = h\, f(t_n+h, y_n+k_1)$$

$$y_{n+1} = y_n + 0.5\,(k_1 + k_2)$$

If we make use of the additional stage:

$$k_3 = h\, f(t_n+h/2, y_n+(k_1+k_2)/4)$$

A. How can we use this additional evaluation to get a third order rule? Hint: think of Simpson's rule!

B. If we use the difference between the two-stage and three-stage rules above to estimate the error at each step, how do you compute the optimum step size for some allowable error = τ ?

C. Will this rule be stable for very stiff ODE's? Briefly explain your answer.

Problem 4. (15 points) Integration Error Propagation:

A. Derive the amplification factor and local error for the Backward Euler integration rule.

B. For what values of the Jacobian is this rule stable?

C. What is the overall order of this quadrature rule?