Cheg 258 Third Hour Exam **5/6/99**
Closed Book and Closed Notes

Problem 1). Integration Error Propagation:

a. Derive the local error and error amplification factor for the Backward Euler method.

b. What is the stability interval for this method?

c. Briefly discuss the relative advantages and disadvantages of the two-stage Runge Kutta method and the Trapezoidal rule for non-linear ODE's. For non-stiff ODE's, which rule is more accurate?

Problem 2). Error Propagation and Statistics:

It is desired to calculate the radius of a sphere by measurement of its mass and knowledge of its density. The density of the material making up the sphere is reported to be:

$$\rho = 1.32 \pm 0.02 \text{ g/cm}^3$$

where the error is one standard deviation. How accurately do we have to measure the mass to get the radius to a precision of ±2% at the 95% confidence level?

Problem 3). Error Propagation / Optimization:

A builder has recently purchased a100 acre tract of land and is planning to build houses on it. She can either subdivide the tract into small 1/4 acre lots and build a large number of small houses, or can use a fewer number of larger 1 acre lots (and houses) which usually have a significantly greater profit margin but are much riskier. We assume that any combination of the large and small lots are possible (not usually a great assumption). Based on past experience with the market the profit on a small house is expected to be \$10k ± 5k/house and the profit on a large house is \$60k ± 80k/house. The covariance between large and small house profits is 175 $(k\$/house)^2$. In a given year all large houses have the same profit, as do all small houses. Since our contractor got burned the last time around, she is fairly risk averse. How many of each size house should she build so that she 1) makes the largest possible expected profit subject to the constraint that 2) she has at least an 85% probability of breaking even?

Problem 4). Systems of Equations:

A common chemical reaction is a liquid catalyzed second order reaction such as is depicted below. Basically, two gases A and B dissolve at the surface of a liquid film of thickness L and react with a rate K c_A c_B where K is a second order rate constant. The species concentrations at the surface are given by the equilibrium concentrations c_{A0} and c_{B0}, while the derivatives of the concentrations at the impermeable inner wall are zero. The concentrations of the species are governed by the differential equations:

$$D \frac{d^2 c_A}{dx^2} - K\, c_A\, c_B = 0 \quad ; \quad D \frac{d^2 c_B}{dx^2} - K\, c_A\, c_B = 0$$

and by the boundary conditions:

$$c_A \big|_{x=0} = c_{A0} \quad ; \quad c_B \big|_{x=0} = c_{B0} \quad ; \quad \left.\frac{dc_A}{dx}\right|_{x=L} = \left.\frac{dc_B}{dx}\right|_{x=L} = 0$$

a). Recast this problem in terms of a system of equations, carefully defining all variables, initial conditions, shooting parameters, etc.

b). What is the Jacobian for this set of equations?

c). Briefly describe how you would determine the stability characteristics of the system.