Why Getting Numerical Analysis Right Matters:

Or a Layman's View of Economic Disaster

Definitions

- CDO: Collateralized Debt Obligation - A mixed up collection of mortgages or other debts (designed to reduce risk).
- Tranche: A way of dividing up a CDO to make pieces less (senior) or more (mezzanine) risky.
- CDO^2: A recombination of mezzanine CDO's into new senior and mezzanine tranches.
- CDS: Credit Default Swap - A weird sort of life insurance for bonds and mortgages (designed to reduce risk).
- Copula: The covariance of default risk.

The CDO Tranches

- Major investors such as pension funds can only invest in AAA bonds, as they are risk averse.
- Subprime mortgages are intrinsically risky (that's why they are subprime!).
- To make an investment grade bond, a bunch of subprime mortgages are combined into a pool, diversifying the risk.
- This pool is then subdivided into *tranches*. Any defaults are assigned to the lowest tranche first (potentially wiping it out). The senior tranches are the last to get defaults.
- The bottom (mezzanine) tranches were then pooled together, and then redivided into tranches again, producing the CDO^2 financial product.
- The top tranches were sold off to banks and pension funds, creating a huge market for subprime loans.

The CDS

- Banks like to control risk by insuring against loss, which is an important way of protecting the financial system.
- CDO's were insured via something called a *credit default swap*.
- Insurance was sold that would pay out if the CDO defaulted, and the insurance company got a premium for their risk.
- Rather than just selling the CDS to the bond owner (which makes sense), AIG and other insurers sold CDS's to anyone, including hedge funds which recognized that the insurers were underestimating the risk.
- This is sort of like taking out a life insurance policy on your neighbor, without even letting him know…
- The total exposure on CDS's was actually far greater than the total CDO market.
- Goldman and deal makers made fees on every transaction.

Risk Estimation

- In order to price the insurance and rate the risk of the CDO's, it was necessary to determine the probability of default.
- Because they were pooled investments, the default rate was critically dependent on the covariance of the subprime mortgages.
- Rather than examining the bonds making up the CDO, David Li proposed that the market price of the related CDS could be used to determine the *copula*.
- This worked great, except all the available data was during a period of rising home prices - when the covariance was naturally small.
- When the housing market started to go down, really bad things happened…

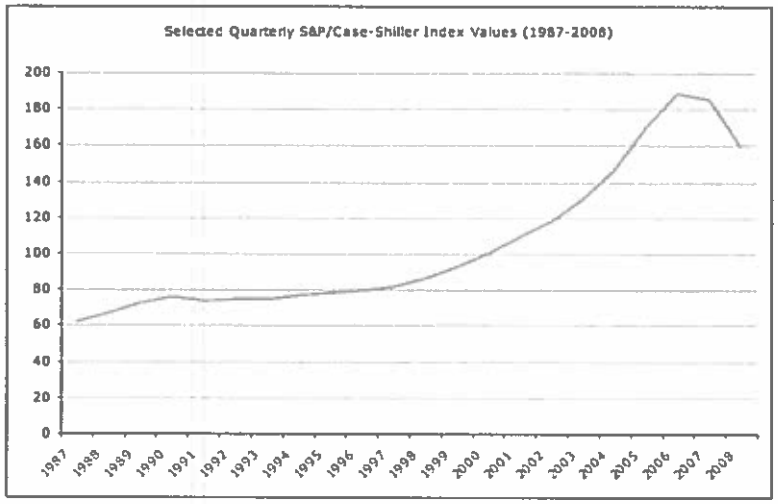
The Effect of Greed!

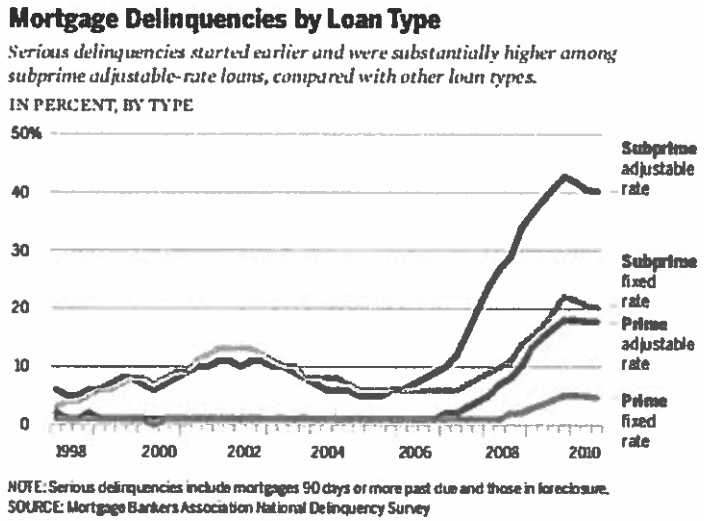
- Wall Street was rewarded via fees for brokering deals.
- Because a market was created for sub-prime (read "bad") loans, lenders solicited wildly inappropriate borrowing and passed the bad loans off to others. This increased the total amount of borrowing far beyond a sustainable level.
- Insurers would issue CDS guarantees in an amount far greater than the CDO being insured: again, they would get lots of fees.
- Provided that housing prices kept going up, the banks and the insurers thought they had a great deal!

Why the error?

- CDO correlation used CDS valuation, only available during a period of rising home prices!
- Covariance is massively underestimated in an adverse housing market.
- Reality was too painful to believe, and easier to ignore.
- Senior management used the "black box" calculations and didn't really understand the implications of the underlying assumptions.

Housing Price Variation





Leverage: Compounding the error

- Leverage means borrowing money to invest in a higher yielding opportunity
- Leverage can be very good, or very very bad.
- You have to accurately asses the risk of the investments
- Financial institutions underestimated the risk of CDO's, so what they *thought* was a sure thing, was really a sure disaster
- At the end, ML was leveraged 30:1 in CDO's, with predictable results…

A Timeline

- 1997 housing price index = 80
- 1997 CDS invented by JP Morgan Chase
- 2000 Paper by David Li "On Default Correlation: A Copula Function Approach" - determined default correlation from CDS pricing
- 2000 "Commodity Futures Modernization Act of 2000"- A "late night" bill deregulating credit default swaps
- 2001 $920 billion in CDS outstanding, $275 billion in CDO's
- 2006 peak housing price index of 190 (2.4x in 9 years!)
- 2006 CDO market hits $4.7 trillion (17x increase in 5 years!)
- 2007 CDS market hits $62 trillion (67x increase in 6 years!)
- 2008 Lehman Brothers declares bankruptcy
- 2009 Troubled Asset Relief Program costs the taxpayer nearly $1T

The Take Home Message:

- Numerical & Statistical Analysis can be very useful to understand complex systems!

- You must -always- question the assumptions that go into any numerical model, particularly one involving evaluation of risk/uncertainty.

- Most systems and models will 1) assume some level of independence, and 2) underestimate the degree of covariance.

- DON'T BELIEVE THE ANSWER JUST BECAUSE THE COMPUTER GIVES YOU A NICE GRAPH OR NUMBER! UNDERSTAND WHAT YOU ARE DOING!

Further Reading

- Michael Lewis, "The Big Short", 2010.
- Felix Salmon, "Recipe for Disaster: The Formula That Killed Wall Street" Wired.com 2009.
- Anna Barnett-Hart, "The Story of the CDO Market Meltdown: An Empirical Analysis" Senior Thesis, Harvard University 2009.

Review of Matrix Ops: (7)

A matrix is a way of representing a large amount of information in a compact way.

Matlab ≡ Matrix Laboratory

=> it's all about manipulating matrices!

Suppose we look at the HW grades for a class. Let:

G_{ij} ≡ grade of the i^{th} student on the j^{th} homework

So the matrix $\underset{\sim}{G}$ might be:

$$\underset{\sim}{G} = \begin{bmatrix} 9 & 8 & 7 & 8 \\ 10 & 6 & 9 & 10 \\ 2 & 3 & 1 & 0 \end{bmatrix}$$

← 1st student

← 2nd student

↑ 1st assignment

↑ 2nd assignment

Suppose we want to calc. the average for each student.

Let $\underset{\approx}{G}$ be an $n \times m$ matrix

(n rows, m columns)

Then the vector $\underset{\sim}{AG}$ is:

$$\underset{\sim}{AG} = \underset{\approx}{G} \cdot \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{\frac{1}{m}} \equiv n \times 1 \text{ vector}$$

$n \times m$ matrix $\leftrightarrow$ $m \times 1$ vector

these must match!

We multiply each row of first matrix by each column of 2nd matrix (only one here)

So $\underset{\approx}{A}\,\underset{\approx}{B} = \sum_j A_{ij} B_{jk} = C_{ik}$

which is a series of nested loops!

In Matlab, this is a lot more compact!

$$\underset{\sim}{AG} = \underset{\approx}{G} * \text{ones}(4,1) * \frac{1}{4}$$

$$\underset{\sim}{AG} = \text{sum}\left(\underset{\approx}{G}'\right)' * \frac{1}{4}$$

(sum command sums over columns! we use ' to take transpose!)

$$\underset{\sim}{AG} = \text{sum}\left(\underset{\approx}{G}, 2\right) \frac{1}{4}$$

↑ forces sum over rows!

All yield the same result!

Final note: Matlab scripts & functions

Run example!

- Break your code into functions and scripts
- comment your code so you can remember it!
- define function I/O & purpose in comment on function (1st line)