CBE 20258 Final Exam **5/9/14**
You may use one sheet of personally prepared notes…

Problem 1). Integration Error Propagation:

a. Derive the local error and error amplification factor for the Backward Euler method.

b. What is the stability interval for this method?

c. Briefly discuss the relative advantages and disadvantages of the two-stage Runge Kutta method and the Trapezoidal rule for non-linear ODE's. For non-stiff ODE's, what is their relative accuracy?

Problem 2). Quadrature:

Accurately estimate the integral:

$$I = \int_0^1 \cosh(x)\ln(x)\,dx$$

using two-point Gaussian quadrature. Show all of your work.

Problem 3). Error Propagation / Optimization:

A builder has recently purchased a 100 acre tract of land and is planning to build houses on it. She can either subdivide the tract into small 1/4 acre lots and build a large number of small houses, or can use a fewer number of larger 1 acre lots (and houses) which usually have a significantly greater profit margin but are much riskier. We assume that any combination of the large and small lots are possible (not usually a great assumption). Based on past experience with the market the profit on a small house is expected to be \$10k ± 5k/house and the profit on a large house is \$60k ± 80k/house. The covariance between large and small house profits is 175 (k\$/house)^2. In a given year all large houses have the same profit, as do all small houses. Since our contractor got burned the last time around, she is fairly risk averse. How many of each size house should she build so that she 1) makes the largest possible expected profit subject to the constraint that 2) she has at least an 85% probability of breaking even?

Problem 4). Systems of Equations:

Next semester in both transport classes you will study the famous Falkner-Skan equation which describes boundary layer flow past a wedge. The equation for the streamfunction f is given by the third order non-linear ODE:

$$\frac{d^3 f}{d\eta^3} + f\,\frac{d^2 f}{d\eta^2} + \beta\left[1 - \left(\frac{df}{d\eta}\right)^2\right] = 0$$

where β is just a constant (the wedge angle is βπ, actually). The boundary conditions at the surface of the wedge and far from the wedge are:

$$f(0) = 0 \quad ; \quad \left.\frac{df}{d\eta}\right|_{\eta=0} = 0 \quad ; \quad \left.\frac{df}{d\eta}\right|_{\eta=\infty} = 1$$

We are interested in calculating the displacement thickness δ* (a measure of how thick the boundary layer is), which is defined by:

$$\delta^* = \int_0^\infty \left(1 - \frac{df}{d\eta}\right) d\eta$$

a. Convert this problem to a system of equations in matrix form, identifying all variables, derivatives, boundary conditions, etc.

b. Show how you would use the techniques described in class to determine the displacement thickness, defining all variables, procedures, and functions used. Include in your description how you best deal with a boundary condition at infinity for this equation (and integral). Note that I don't want you to actually write the code, but I want you to show me that you know how to do it!