Cheg 258 First Hour Exam **2/23/96**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

Problem 1. Gaussian Elimination Error:

a. The -orthogonal- matrix **A** is given by:

$$\mathbf{A} = \begin{matrix} -0.3015 & 0.8616 & 0.4082 \\ -0.9045 & -0.1231 & -0.4082 \\ -0.3015 & -0.4924 & 0.8165 \end{matrix}$$

What is its 1-norm condition number? Show your work.

b. Prove that the 2-norm of an orthogonal matrix is unity.

Problem 2. Finite Differences

A finite difference approximation for the second derivative of a function f(x) which is defined only for $x \geq x_0$ (e.g., up to the left edge of a domain) is given by:

$$f''(x_0) \approx [f(x_0) - 2 * f(x_0 + h) + f(x_0 + 2h)] / h^2$$

a). What is the order in h of the algorithm error of this approximation?

b). Combining this algorithm error with numerical error in the calculation of f(x) determine the approximate optimum value of h and the minimum possible error. We are not worried about constants here, but I want to know how they depend on the machine error ε and the function and its derivatives.

Problem 3. Systems of Equations:

Remember the word problems you used to get in your grade school math courses? These puzzles can often be set up as a series of linear equations. Consider the following problem:

A family has three children, George, Sally, and Fred. If the sum of their ages is 27, Fred is four years older than Sally, and George is half as old as the sum of Fred and Sally's ages, what are the ages of the three children?

Set this problem up as a 3x3 matrix problem of the form **A x** = **b** and **solve it using gaussian elimination**. Use the back of this sheet as well if you need more space.

Problem 4. Linear Regression:

A stagnation flow towards a plate (the flow you get if you squirt a jet of water at a flat surface) causes the particles in the water to follow the trajectory:

$$y = \frac{y_0}{1 + \beta\, y_0\, t}$$

where y_0 is the initial position of the particles (distance from the plate) and β is a measure of the strength of the flow. We want to calculate β using linear regression from the data:

t(sec)	y (μm)
1	1.5
2	0.8
3	0.6
4	0.4

Set the problem up in the matrix form used in class, explicitly identifying **A** and **b**, and providing definitions for the elements of **x**.

Hint: you will have to rework the model so that it is linear in the modelling parameters.