Cheg 258 Second Hour Exam **4/3/96**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. You may use the backs of the sheets as well. Please the blue books as scratch paper only!

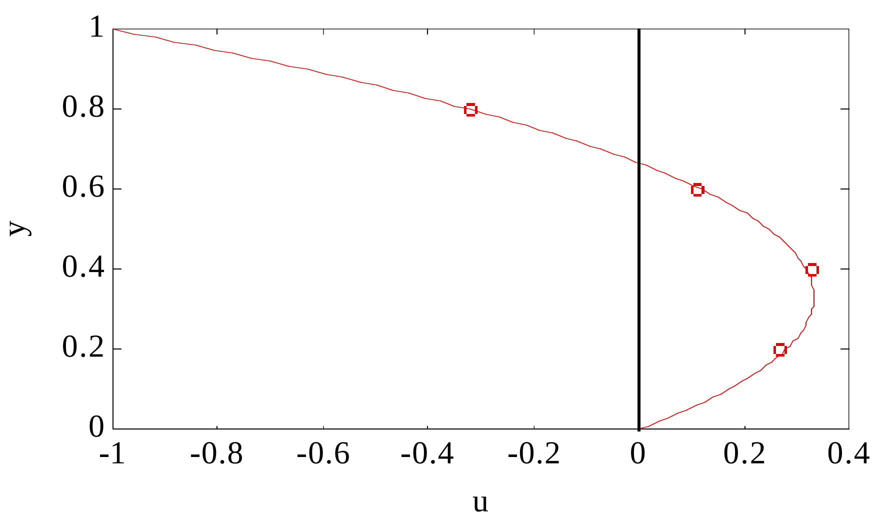
Problem 1). Regression and Error Analysis:

In an experiment done in my laboratory in undergraduate research a few years ago we looked at particle motion in Taylor-Dean flow. We won't worry about the details here, but we will examine this flow field. Suppose we have a velocity distribution which is described by the quadratic function:

$$u = x_1 y + x_2 y^2$$

where x_1 and x_2 are parameters characterizing the velocity profile. Using a probe, we measure the velocity at the following positions:

y	u
0.2	0.27
0.4	0.33
0.6	0.11
0.8	-0.32



A. We are interested in determining the point in the interior of the flow where the velocity is zero. Show how you would calculate this, and state any assumptions that you make. Explicitly define any matrices you use in the calculation.

B. What is the error in this calculated value? Again, state all of your assumptions. I am not looking for a number, but rather I want to see all of the equations you use, and for you to be as specific as possible. Define all matrices as explicitly as you can.

Problem 2). Multidimensional Root Finding:

In this problem we examine Newton's method for finding the root to a set of equations.

A. Suppose we have a set of N non-linear equations in N variables. Derive Newton's method for finding the roots to these equations.

B. Use this technique to solve the pair of equations:

$$f_1 = x_1^2 + x_2^2 - 5 = 0$$
$$f_2 = x_1 x_2^2 - 2 = 0$$

Start with the initial guess $x = (1, 1)^T$ and do one iteration. Be sure to define all of the matrices used in your calculations!

Hint: The inverse of the 2 x 2 matrix [a, b; c, d] is given by [d, -b; -c, a]/(ad - bc).

Problem 3). Optimization:

We examine the planning of an orchard of cherry trees. Suppose we have \$20,000 to invest in planting cherry trees. We can plant a mix of sour (dessert) cherries and sweet cherries (the ones you like to eat). The sour cherry trees cost \$20 each, and the sweet cherry trees cost \$10 each. The sweet cherries, however, can be sold for \$100 per tree per year when mature if fully pollinated, while the sour cherries can be sold for only \$25 per tree when mature. Finally, the yield per tree of the sweet cherry trees depends on their proximity to the sour cherry trees, since they are not self-pollinating. If the number of sour trees is given by x_1 and the number of sweet trees is x_2, we shall take the yield (the fraction of the fully pollinated yield) of the sweet trees to be $x_1 / (x_2 + x_1)$.

Show how you would calculate the optimum number of sweet and sour cherry trees. Be as specific as possible, defining all functions and techniques employed.