Cheg 258 Second Hour Exam **4/14/99**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

Problem 1. Root Finding.

A. Derive Newton's method for solving the **system** of equations $\underset{\sim}{f}(\underset{\sim}{x}) = 0$.

B. Apply Newton's method to solve the pair of equations below. Do one iteration from the given initial guess. It is probably easier to solve for the correction using Gaussian elimination than it is to explicitly take the inverse of the Jacobian. Show your work.

$$x_1^2 + x_1 x_2 = 3 \qquad \underset{\sim}{x}^0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$x_1 + x_2^2 = 10$$

Problem 2. Quadrature and Error:

A laboratory experiment requires measurement of the average temperature of a reactor over its width, e.g. the integral:

$$\overline{T} = \frac{1}{H}\int_0^H T(x)\, dx$$

It turns out that because of the experimental geometry it is possible to make measurements only at the surface $x = H$ and at one point in the interior. You are asked to determine the optimal location for making the interior measurement (e.g., where on the domain $0 < x < H$ they should drill the hole for the thermocouple) and to determine the corresponding quadrature weights for both node locations. What is the answer to this problem which will yield the highest polynomial degree rule?

Hint: The optimal node location is <u>not</u> symmetric for this problem!

Problem 3. Optimization and Error:

A farmer is trying to determine what crops to plant in his fields for the upcoming year. He has decided to plant either corn or wheat, or a mixture of both. The expected return for either crop is the same per acre, but from his experience there is a significant variability in this return which he would like to minimize. Analysis shows that the returns are characterized by:

$$R_c = 500 \pm 100 \ \$/\text{acre} \qquad \text{covariance} \approx \frac{1}{2}\sigma_{R_c}\sigma_{R_w}$$
$$R_w = 500 \pm 80 \ \$/\text{acre}$$

How should the farmer divide up his acreage to get the minimum uncertainty in his return?

Problem 4. Regression and Error:

In Senior Lab students measure the natural convection due to a heated wire. Basically, the wire heats the fluid, the fluid expands, and the fluid in the vicinity of the wire rises due to buoyancy forces - just like the draft rolling off a cold window. Theory predicts that the velocity directly over the wire should increase very slowly as we move higher according to the power law relationship:

$$u = C\, x^{s}$$

where the exponent s has a theoretical value of 0.2. A student has made the following set of measurements:

height	velocity
1	0.99
2	1.15
3	1.28
4	1.31

Explain how the student could answer the question of whether her power law exponent matched theory to within experimental error at the 95% confidence level. I want you to define all matrices, assumptions, etc. You can also give me the final answer (e.g., crunch the numbers) if you wish (and if you have a fancy enough calculator), but I am principally interested in you demonstrating that you know how to solve the problem.

(one point extra credit) There are 34 students in the class, and questions of the day are asked at random. If we ask a total of 80 questions of the day, what fraction of the class will never be called?