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Cheg 258 Third Hour Exam **5/7/96**
Closed Book and Closed Notes

Problem 1). Integration Error Propagation:

a. Derive the local error and propagation error for the Euler method.

b. What is the stability interval for this method?

c. Name two high order (e.g., greater than O(h)) methods we have discussed in class useful for stiff ODE's.

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Problem 2). Runge-Kutta Methods:

The differential equation:

$$\frac{d\,y}{d\,t} = -\,4\,y, \quad y(0) = 1$$

has the solution:

$$y = e^{-4\,t}$$

Solve this equation numerically using the 2-stage Runge-Kutta method with step size h=1/2 over the interval [0,1] (e.g., for two steps) and compare your result to the exact answer. Why do they differ so greatly?

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Problem 3). Weighted Linear Regression:

We have made a series of observations b_i such that we know the variance of each point, given by $\sigma_{b_i}^2$. These observations are all **independent**, but the variances **are not the same!!**

a. Write down the form of the matrix of covariance of the observations (this is easy - don't make it hard).

b. The weighted regression problem may be stated as:

$$\min_{\underset{\approx}{x}} \ \underset{\sim}{r}^T \left(\underset{\approx}{\Sigma}_x^2\right)^{-1} \underset{\sim}{r}$$

where:

$$\underset{\sim}{r} = \underset{\approx}{A}\, \underset{\sim}{x} - \underset{\sim}{b}$$

Develop (or derive) the normal equations for the weighted regression problem.

c. Determine the matrix of covariance for the fitting parameters $\underset{\sim}{x}$.

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Problem 4). Quadrature:

Accurately estimate the integral:

$$I = \int_0^{\pi} \cos(x) \ln(x)\, dx$$

using two-point Gaussian quadrature. Show all of your work.