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Cheg 258 Third Hour Exam **5/6/98**
Closed Book and Closed Notes

Problem 1). Integration Error Propagation:

a. Derive the local error and error amplification factor for the Euler method.

b. What is the stability interval for this method?

c. Briefly discuss the relative advantages of Gear's method and the 4-stage Runge-Kutta method.

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Problem 2). Error Propagation and Statistics:

It is proposed that you estimate the radius of a small sphere by measuring how fast it falls through a viscous fluid. The velocity is given by the relation:

$$U = \frac{2}{9}\frac{(\rho_s - \rho_f)\, g\, a^2}{\mu}$$

where U is the velocity, ρ_S is the density of the solid, ρ_f is the density of the fluid, g is the acceleration due to gravity, a is the radius, and μ is the viscosity. The measurements, with their 1σ error (uncertainty) are given below:

$$U = 0.050 \pm 0.005 \text{ cm/s}$$
$$\rho_S = 1.30 \pm 0.08 \text{ g/cm3}$$
$$\rho_f = 1.07 \pm 0.02 \text{ g/cm3}$$
$$g = 980 \text{ cm/s2}$$
$$\mu = 10.1 \pm 2 \text{ g/(cm s)}$$

a. Calculate the radius together with its 95% confidence interval.

b. Which of the measurements would you first try to improve in order to get a more accurate measurement of the radius? Briefly explain your answer.

c. If I repeated the velocity measurement (only velocity, none of the others) ten times, by what factor would the uncertainty in the new average velocity be reduced, and by what factor would the uncertainty in the calculated radius be reduced?

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Problem 3). Error Propagation / Optimization:

a. You have been given $10,000 as a graduation present. You decide to invest it for a year, and then spend it. You want to maximize the return on your investment, but you also don't want to lose any money either. You can either invest it in a 1 year CD at 5% guaranteed rate, or put it in a mutual fund which has an expected return of 10% but an uncertainty (1 σ) of 10% too. What is your optimum investment strategy if you require a 97.7% probability that you at least break even?

b. How does the answer change if you use a 2-year investment horizon with a 50% covariance from year to year (e.g., $\sigma^2_{S_1S_2} = 0.5\,\sigma_{S_1}\,\sigma_{S_2}$ where $\sigma_{S_1} = \sigma_{S_2} = 0.1$ is the standard deviation of the mutual fund in each year)? You may ignore the effect of compounding if you wish.

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Problem 4). Systems of Equations:

a. A plug flow reactor is governed by the second order non-linear differential equation:

$$U\,\frac{\partial c}{\partial x} = D\,\frac{\partial^2 c}{\partial x^2} - K\,c^2 \;;\; c\,(0) = 1,\ \frac{\partial c}{\partial x}\,(0) = 0$$

where U is the fluid velocity, c is the concentration, D is the diffusion coefficient, and K is the reaction rate of this second order (non-linear) reaction. Convert this equation into a system of first order differential equations and appropriate boundary conditions.

b. Determine the stability characteristics of these equations.

c. Under what conditions will an explicit numerical integration procedure be stable for this problem?