## CBE 30356 TRANSPORT PHENOMENA II

Mid-Term Exam
3/3/22
You may have one sheet of hand written notes for this exam.
Please solve the exam on the pages provided. If you need more space, please use a blue book, but make sure you put your name on it! If you separate the pages, make sure your name is on each one too!

Honor Code:
As a member of the Notre Dame community, I acknowledge that it is my responsibility to learn and abide by principles of intellectual honesty and academic integrity, and therefore I will not participate in or tolerate academic dishonesty.

Problem 1 (16 points). Dimensionless Groups: A recurring emphasis of this class is the use of dimensionless groups of parameters for describing phenomena. Listed below you will find 8 such groups we've used this term. For each of these, enter the letter corresponding to the correct physical description and name from the list at the bottom of the page.

| question | Dimensionless <br> group | Definition | Name |
| :--- | :--- | :--- | :--- |
| 1 | $\frac{U D}{v}$ | $\frac{U D}{\alpha}$ | $\frac{h D}{k}$ |
| 2 | $\frac{h_{e x t} R}{k_{\text {int }}}$ | $\frac{v}{\alpha}$ |  |
| 3 | $\frac{h}{\rho C_{p} U}$ |  |  |
| 4 | $\frac{\tau_{w}}{\frac{1}{2} \rho U^{2}}$ |  |  |
| 5 | $\left(\frac{\omega a^{2}}{v}\right)^{1 / 2}$ |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 8 |  |  |  |

Definitions: a. $\frac{\text { Heat Transfer }}{\text { Convective Heat Transfer }}$, b. $\frac{\text { Heat Transfer }}{\text { Conductive Heat Transfer }}$, c. $\frac{\text { Inertial Forces }}{\text { Viscous Forces }}$,
d. $\frac{\text { Convection of Energy }}{\text { Diffusion of Energy }}$, e. $\frac{\text { Wall Shear Stress }}{\text { Dynamic Pr essure }}$, f. $\frac{\text { Tube Radius }}{\text { Penetration Length }}$,
g. $\frac{\text { Momentum Diffusivity }}{\text { Energy Diffusivity }}$, h. $\frac{\text { Internal Heat Transfer Re sistance }}{\text { External Heat Transfer } \text { Re sistance }}$

Names: a. Prandtl Number, b. Fanning Friction Factor, c. Stanton Number, d. Womersley Number, e. Nusselt Number, f. Biot Number, g. Reynolds Number, h. Thermal Peclet Number

Problem 2. (20 points) Heat Conduction in Solids: Some of your classmates have been working on aspects of heat transfer in a melter/blender for HB Fuller. A key issue in improving the rate of heat transfer (and speeding up the processing time) is evaluating the heat transfer coefficient -inside- the mixer, rather tricky to do from the outside! It is proposed to make a "heat transfer stethoscope" as shown below. A sandwich (internal thickness d) is attached to the wall of the vessel, and the temperature inside the vessel and on both surfaces of the sandwich are measured (the outer part of the sandwich would have a controlled temperature).
a. If the three temperatures are $\mathrm{T}_{\mathrm{M}}, \mathrm{T}_{1}$, and $\mathrm{T}_{0}$ as shown, and the thermal conductivity of the material of thickness $d$ making up the sandwich is $k$, derive an expression for the internal heat transfer coefficient on the inside of the vessel.
b. The internal heat transfer coefficient could be a function of time. If the volumetric heat capacity of the material making up the sandwich is $\rho C_{p}$, estimate the response time of the probe.

h, $\mathrm{TM}_{\mathrm{M}}$

Problem 3. (20 points) Convective Heat Transfer: Consider the geometry depicted below. An upper plate at $\mathrm{y}=\mathrm{d}$ is moving with a velocity U in the x direction, generating a simple shear flow between the two plates. The fluid is heated for all $x>0$ with a constant heat flux $\mathrm{q}_{0}$, and the upper surface is insulated (so the fluid is getting hotter with $x!$ ). We are interested in the asymptotic solution and Nusselt number.
a. Write down the equations and boundary conditions governing this problem and render them dimensionless.
b. Solve for the asymptotic temperature distribution at large $x$.
c. Using this, calculate the dimensionless flow average temperature and temperature at the wall.
d. Using these two values, calculate the asymptotic Nu for these boundary conditions.


Name
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Problem 4. (20 points): Boundary Layer Solution. The local Nu calculated in problem 3 is only valid for large $x$. For small $x$ it is more appropriate to look at the boundary layer where energy hasn't had a chance to diffuse very far from the lower wall. In this problem we examine this limit.
a. Rescale the equation and boundary conditions in this boundary layer limit, rendering the problem dimensionless.
b. From scaling (or from affine stretching) determine how the wall temperature depends on $x$.
c. Recognizing that the bulk temperature in this limit is just $T_{0}$ (e.g., the boundary layer is thin!) determine the local Nu as a function of x to within an $\mathrm{O}(1)$ constant. Note that I'm not asking you to -get- the constant, as that would require solution to the transformed ODE (which I'm also not asking you to get). Scaling alone gets you almost all the way there!

