CBE 30356 TRANSPORT PHENOMENA II

Mid-Term Exam
3/9/23
This exam is closed books and notes.

Problem 1 (16 points). Dimensionless Groups: A recurring emphasis of this class is the use of dimensionless groups of parameters for describing phenomena. Often we can regard dimensionless groups as ratios of physical mechanisms. For each of the given ratios, determine the corresponding group of parameters from the list below. Note that there are a few "distractors" that won't appear!

a. $\dfrac{Heat\ Transfer}{Convective\ Heat\ Transfer}$

b. $\dfrac{Heat\ Transfer}{Conductive\ Heat\ Transfer}$

c. $\dfrac{Inertial\ Forces}{Viscous\ Forces}$

d. $\dfrac{Convection\ of\ Energy}{Diffusion\ of\ Energy}$

e. $\dfrac{Wall\ Shear\ Stress}{Dynamic\ \mathrm{Pr}essure}$

f. $\dfrac{Tube\ Radius}{Penetration\ Length}$

g. $\dfrac{Momentum\ Diffusivity}{Energy\ Diffusivity}$

h. $\dfrac{Internal\ Heat\ Transfer\ \mathrm{Re}sis\tan ce}{External\ Heat\ Transfer\ \mathrm{Re}sis\tan ce}$

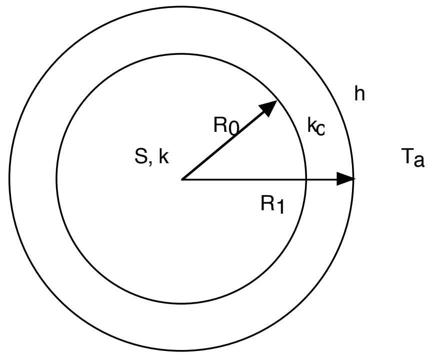
Possible parameter ratios:

$$\frac{UD}{\upsilon},\ \frac{h_{ext}R}{k_{\mathrm{int}}},\ \frac{\tau_w}{\frac{1}{2}\rho U^2},\ \frac{U^2}{Lg},\ \left(\frac{\omega a^2}{\upsilon}\right)^{1/2},\ \frac{\upsilon}{\alpha},\ \frac{UD}{\alpha},\ \frac{hD}{k},\ \frac{h}{\rho C_p U},\ \frac{\upsilon}{D_{AB}},\ \frac{\mu\dot{\gamma}a}{\Gamma}$$

Problem 2. (20 points) Heat Conduction in Solids: Nuclear waste disposal is an important problem, complicated by the high level of radioactivity in a spent fuel rod that generates a lot of heat! After cooling in a pond for 5 to 20 years, sometimes these spent rods are transferred to a "dry cask" such as depicted below. The radioactive material, with heat source/volume S, radius R_0, and conductivity k is surrounded with a sealed cask of conductivity k_c and outer radius R_1. If the outside of the cask has a heat transfer coefficient h with the surroundings at temperature T_a, your goal is to figure out the temperature at the center of the radioactive energy source. We'll simplify this problem by ignoring any conduction in the z (or θ) direction.

a. Using an energy balance, determine the total energy release (per unit length in the z direction).

b. What is the temperature at the outer edge of the cask?



c. What is the temperature at the inner edge of the cask?

d. What is the temperature at the center of the radioactive heat source?

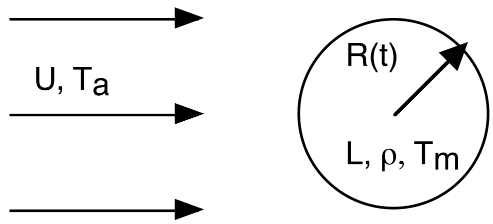
The equation of energy with source in cylindrical coordinates is:

$$\rho C_p\left(\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

and

$$q_r=-k\frac{\partial T}{\partial r};\quad q_\theta=-k\frac{1}{r}\frac{\partial T}{\partial \theta};\quad q_z=-k\frac{\partial T}{\partial z}$$

Problem 3. (20 points) Convective Heat Transfer/Scaling: A sphere of ice of radius R_0 is suspended in a stream of air with velocity U and temperature T_a. Since $T_a > T_m$ (the melting temperature of the ice), the heat flux will cause the ice to melt, reducing the radius in time. This is a slow process, however, so we can use pseudo steady-state to figure out the melting rate. The melting process also causes the entire sphere of ice to be at an unchanging temperature T_m until it is all gone, thus all heat transfer is external. We are also going to ignore both evaporative and condensation heat transfer of water (this is reasonable if the dew point of the air stream is at T_m). The latent heat of fusion of the ice is L (energy/mass) and the density is ρ.



The problem is essentially governed by the external heat transfer coefficient h. While many correlations can be used, for this problem assume that the Nusselt number is given by the correlation suggested in BSL at high Re (note that the diameter is twice the radius and changes in time!):

$$Nu \approx 0.6\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

a. Using an energy balance, write down the equation which describes the change in radius with respect to time.

b. Rendering R dimensionless with respect to R_0 (the initial radius) and t with respect to t_c, determine the scaling for the melt time, and how it depends on the parameters of the problem.

c. Solve the dimensionless differential equation to determine how long it takes for the ice to melt (e.g., get the O(1) number…).

d. Suppose we humidify the air a bit (without changing the temperature or other material properties). Would this increase or decrease the melt rate? *Briefly* justify your answer.

Problem 4. (20 points): Transient Boundary Layers. A semi-infinite slab initially at a temperature T_0 is heated at y = 0 with a heat flux $q_y \mid_{y=0} = A\, t$ (e.g., the heating increases linearly in time).

a. Write down the equation and boundary conditions which govern this problem.

b. Render the equations dimensionless, determining appropriate temperature and length scales in terms of the unknown time scaling.

c. Using the result of the scaling (or using affine stretching) show that the problem admits a self-similar solution, obtaining the similarity rule and variable in canonical form. You don't need to get the transformed ODE.

d. What is the temperature of the lower plate as a function of time (to within an unknown O(1) constant….).